Teaching Portfolio

Hitesh Gakhar

November 21, 2022

This teaching portfolio contains sample materials such as worksheets, quizzes, homework assignments, review problems, and exams, and student feedback from selected courses that I have taught at the University of Oklahoma and at Michigan State University. The organization is as follows:

Contents

For more information about any of these courses, please feel free to contact me at hiteshgakhar@ou.edu.

1. Teaching Statement

1.1. Introduction

I enjoy teaching and communicating mathematics, with a focus on the underlying implicit skill of reasoning. My general teaching strategy is student-focused because students in different courses grasp ideas differently even when they are centered around the same theme. During my time at the University of Oklahoma (OU) and Michigan State University (MSU), I have taught a wide range of courses as the instructor of record, including Linear Algebra, Transitions to Proofs, Differential Equations, the entire sequence of Calculus, and Survey of Calculus-I. At OU, I have served as an advisor to two undergraduate students for the *Honors Research Course*, the outcome of which becomes the Undergraduate Honors Thesis. Over 19 semesters, teaching 30 different sections and mentoring dozens of novice graduate instructors, I have worked hard to develop my teaching practice in all formats: traditional lecture classrooms, active learning classrooms, and flipped classrooms. While my work was recognized through the MSU Mathematics TA Award for Excellence in Teaching in 2019, I plan to continuously evolve my teaching practice to make it more inclusive.

1.2. Teaching strategies and experiences

Flipped Classrooms: My first teaching assignment at OU was an online course on multivariable calculus in Fall 2020, at the height of the COVID-19 pandemic. Due to the lengthy class sessions (75 minutes), I was concerned about the attention span of students over Zoom, and hence the efficiency of a regular lecture-style course. This prompted me to switch to a mode that is designed to be more engaging: *Flipped Classrooms.* I made videos for each week that my students were supposed to watch before the class sessions which were geared towards problem-solving and fine-tuning concepts. More specifically, a typical class session would start with taking an unofficial survey on whether the new videos were watched or not, followed by a 5-minute discussion about the new material before we dove into the worksheets. The first problem on a new topic would be solved in an interactive manner on the tablet together with all my students. After a little experience, I would assign a problem, give them time, and then discuss it again keeping everyone involved. In the end, I was very pleasantly surprised at how well it turned out. The students stayed active during discussions, were able to solve the assigned problems, and retained more information than I expected them to. In semesters thereafter, I reused the flipped format for both online and in-person courses in multivariable calculus and linear algebra. I stuck to my basic strategy, making adjustments wherever necessary, and had successful semesters for the students and myself.

> " *. . . The course style, although flipped was actually extremely beneficial to my learning. Viewing the lectures outside of class time and working on problems in class allows for more time to ask questions and really understand the content that is being presented throughout the semester . . .*"
>
> – a multivariable calculus student, Fall 2021

Active Learning and Structured Groupwork: While my lectures always have had an interaction component, I first used active learning techniques in the Transitions to Proofs course at MSU in 2018. In a usual class, I gave a short lecture covering the basics of a topic which was followed by structured student groupwork. The purpose of groupwork was to provide them with in-class hands-on experience, teach them to collaborate, and build interpersonal communication skills. The groups were divided randomly in the beginning, and more strategically as the semester went on; in particular, according to the comfort levels and preferences of the students. Throughout these groupwork sessions, my goal was to help them construct the argument. Although a few students disliked it in the beginning, they agreed that it was beneficial towards the end of the semester.

Traditional Lecturing: During my time at MSU, I lectured for Survey of Calculus-I, the entire Calculus sequence, and Differential Equations. To keep things lively, I would break the lectures up with time for students to work on problems. In the examples that I presented on the board, my intention was to involve the students as much as possible — a strategy that I have stayed with through all my course style and modality changes. It can also be described as helping the students help me drive the solutions forward.

1.3. Teaching beliefs

Accommodation and Access: One very crucial way in which COVID-19 impacted my teaching was that it made me change how I view students and learning. I became a lot more sympathetic to students and their problems. It dawned upon me that life is happening all the time to students as well and more often than not, they are trying their best to survive. Consequently, I became more accommodating with my deadlines, make=ups, and my requirements of documentation needed in these circumstances. I believe that my goal as an educator is to have students master the topic at hand, and if that can be empowered with an extension, I view it as an absolute win. In this spirit, I also make all my materials accessible to my students: *notes, solutions, and pictures of classwork* for anyone who could not make it to class for whatever reason.

Communication and Feedback: Teaching needs to be adapted to the needs of students. The same strategy doesn't work for everyone. Some students prefer listening, while others like a more hands-on approach. These two endpoints create a spectrum of strategies and in my experience, the optimal teaching strategy changes with every batch of students. I use consistent feedback to gauge the atmosphere and adjust my teaching accordingly, which has a two-fold benefit: conflict resolution, and the creation of a positive class experience for the students, which further leads to a better class dynamic. I use feedback at three levels. The first is a *Welcome Survey* sent out to students in or before the first week of the semester asking them a variety of questions about their expectations and concerns about the course.[1] Next, the *day-to-day feedback* is conducted multiple times in every class via a simple show of hands. It involves inquiring if they understood the material, and sometimes, asking them to convince me that they do. This allows me to make spontaneous course corrections: for example, if I observe that the topic in hand is causing trouble, I make room for more examples or discussion by pushing back the next thing on the agenda. I also conduct more thorough *Canvas surveys* several times throughout the semester, often for extra credit.[2]

Methodology and Logic: One of the most important ideas I communicate through my teaching is to prioritize methodology and logic over just the answer. For example in Linear Algebra, students learn a variety of different algorithms, most of them reliant on the row-reduction process which they learn in the first week. While most of them know how to successfully solve a problem using a row-reduction based algorithm, there is an inclination to overlook the underlying ideas, theorems, and the associated mathematical beauty. This can make the students take things for granted, leading to a weaker understanding of the material. To combat this, I use a two-step strategy: first, during my lectures, I often recall theorems and older results while presenting example problems in an attempt to show how connected everything is, and second, while grading I focus more on the ideas than the arithmetic involved.

1.4. Teaching leadership and Undergraduate mentoring

I held several leadership and mentoring positions in the mathematics department at MSU. In 2017, I served as the overall supervisor/coordinator for the courses taught by graduate students in the second half of the summer. I was also a Lead Teaching Assistant for the *Mathematics Learning Center* and the *Center for Instructional Mentoring,* where I mentored and supervised both undergraduate and graduate teaching assistants for several semesters. In my experience, there is always an overlap in the problems different instructors encounter and discussions aid the solution-finding process. At both MSU and OU, I have performed non-evaluative observations that lead to discussions on teaching.

At OU, I have also been an advisor to two undergraduate students, one of them current, for the *Honors Research Course*. The outcome of this course is the students' Undergraduate Honors Thesis. Both students learned about the basics of computational topology and performed simple experiments in the subfield of topological time series analysis in Python.

[1]The survey can be found on: `https://www.hiteshgakhar.com/teaching`

[2]I have used Google forms and paper surveys in the past.

1.5. Diversity and Inclusion

Mathematics, as a discipline, is predominantly male-dominated. This discrepancy in gender ratio is also reflected in the undergraduate classes I teach. Such scenarios can be intimidating to female students, and more generally, minorities. My goal every semester is to create a learning environment where students can express their ideas, even if incorrect, without any hesitation. Being from a different educational system in India, I strongly believe that teaching and learning mathematics is heavily dependent on culture. This often leads to variation in student exposure to mathematics and educational opportunities. I try to foster an environment where students do not get penalized due to these differences.

> " *. . . I have both a physical and learning disability and I truly believe that Dr. Hitesh has the best course structure for those like me to succeed . . .*"
>
> – a linear algebra student, Spring 2021

1.6. Concluding Remarks

In the last eight years, I have learned a lot as an educator. I have improved my communication skills, learned the importance of feedback, and have made my classes more inclusive. I plan to keep improving my teaching methods and make my classes more accessible to all students by being involved in the teaching community at my new institution and seeking out feedback from my peers.

2. Teaching Experience

2.1. OU: Courses taught

1. **MATH 3333: Linear Algebra I**, Fall 2022, Spring 2022, Spring 2021
   Core Topics: Linear Systems, Matrix Algebra, Eigenvectors and Diagonalization, Linear Transformations, Euclidean subspaces, Abstract Vector Spaces
2. **MATH 2443: Calculus & Analytic Geometry IV**, Fall 2021, Fall 2020
   Core Topics: Multivariable Differentiation, Tangent Planes, Gradient, Optimization, Double and Triple Integration, Integration in polar, cylindrical, and spherical coordinates, Divergence, Curl, Green's theorem, Stoke's theorem, Divergence theorem

2.2. MSU: Courses taught

The semesters that precede a (TA) indicate that I was a Teaching Assistant or Recitation Instructor in those semesters.

1. **MTH 299: Transitions (to proofs)**, Fall 2019, Fall 2018, Summer 2018, Spring 2017 (TA)
   This course introduced students to proof-writing in mathematics and included logic and set theory, proof techniques such as contradiction and induction, and applications to real analysis and number theory.
2. **MTH 235: Differential Equations**, Summer 2016, Spring 2016
   This course introduced students to separable and exact equations, linear equations and variation of parameters, higher order linear equations, Laplace transforms, and systems of first-order linear equations.
3. **MTH 234: Calculus III**, Summer 2017, Fall 2016 (TA)
   This course introduced students to vector geometry, multivariable differentiation, double and triple integration in different coordinate systems, and key theorems like Green's, Stoke's and Divergence.
4. **MTH 133: Calculus II**, Spring 2019, Fall 2015 (TA), Spring 2015 (TA), Fall 2014 (TA)
   This course introduced students to one variable integration, sequences, series, parametric equations, and polar coordinates.
5. **MTH 132: Calculus I**, Summer 2019
   This course introduced students to one variable differentiation, optimization, antiderivatives, and FTOC.
6. **MTH 124: Survey of Calculus I**, Summer 2015
   This course introduced students to the idea of limits, continuous functions, derivatives, integrals and their applications.

2.3. Teaching Leadership & Mentoring

1. **Overall Course Coordinator**, Summer 2017
   Duties: Supervising instructors teaching 100-level and 200-level courses, One on one mentoring for a first time Graduate Teaching Assistant, Observations & feedback
2. **Lead Teaching Assistant for Center of Instructional Mentoring**, Spring 2019, Spring 2018, Spring 2017, Fall 2016
   Duties: Observing Graduate Teaching Assistants for Calculus III and Differential equations, Providing feedback, One on one mentoring, Conducting exam reviews for College Algebra
3. **Lead Teaching Assistant for Mathematics Learning Center**, Spring 2019, Spring 2017, Fall 2016, Spring 2016
   Duties: Supervising the Mathematics Learning Center, Observing Undergraduate and Graduate tutors
4. **Teaching Observation Assistant**, Fall 2019, Fall 2018, Fall 2017
   Duties: Observing Undergraduate Learning Assistants & Graduate Teaching Assistants, Providing feedback

3. Sample Syllabi

The next 3 pages contain my syllabus for the semester Fall 2022. I am teaching an undergraduate course in linear algebra. This course introduces students to matrix algebra and vector spaces.
The 4 pages after that contain my syllabus for the semester Fall 2021. I taught an undergraduate course in multivariable calculus and introduced students to multivariate differentiation, integration, and calculus.

DEPARTMENT OF MATHEMATICS

COURSE INFORMATION FOR MATH 3333–002 & 003

Linear Algebra I

Fall 2022

Instructor: Dr. Hitesh Gakhar
E-mail: hiteshgakhar@ou.edu
Office: 1008 PHSC

Office Hours: **M** 4:00 PM–5:00 PM, **F**: 4:00 PM–5:00 PM, and **by appointment**

WEEKDAYS	TIME	ITEM	LOCATION
Monday, Wednesday, Friday	8:30 AM - 9:20 AM	3333–002 Class	PHSC 0323
Monday, Wednesday, Friday	9:30 AM - 10:20 AM	3333–003 Class	PHSC 0321
Monday–Thursday	9:00AM AM-5:00 PM	Math Center	
Friday	9:00 AM- 3:00 PM	Math Center	

For more about the Math Center, visit https://www.ou.edu/cas/mathcenter

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Text: *Linear Algebra with Applications,by W. Keith Nicholson, 2019 version.* **Please note that my lecture notes will be very loosely based on the book. The book is freely available and you can find it under Canvas Modules.**

About the course: This course on introductory linear algebra will be taught in a flipped setting. I will be recording and uploading a few videos before each week. You would be expected to diligently **watch the videos before coming to class** where we would spend our time working on problems together and fine-tuning the concepts. *Some of these classes might start with a pop quiz based on the videos as a form of extra credit.*

Expectations: I expect you to watch videos or read accompanying slides/notes before the class session. I also expect you to attend as many lectures as you can. I expect you to ask lots of questions. I expect you to frequently visit office hours or the Math Center. Above all, I expect you to be an engaged learner. By this I mean that you will engage your peers in and out of class, and ask plenty of questions of me.

Tentative Course Schedule: A tentative course schedule will be available separately.

Grading: While there maybe some curving towards the end of the semester, as of now the following the grade thresholds:

- 90%–100% will be an A or a 4.0
- 80%–90% will be a B or a 3.0
- 70%–80% will be a C or a 2.0
- 60%–70% will be a D or a 1.0
- 0%–60% will be an F or a 0.0

The grade breakdown will be as follows:

H.W.	QUIZ	EXAM 1	EXAM 2	EXAM 3	FINAL	TOTAL
20%	10%	15%	15%	15%	25%	100%

Homework: There will be problem sets (almost) each week on Canvas, which you will upload to Gradescope. *Tentatively*, there will be 11 or 12 written homeworks and the lowest 3 will be dropped.

Quizzes: There will be a quiz *almost* each alternate week. *Tentatively*, there will be 6 quizzes, and the lowest one will be dropped.

Exams: The three midterm exams are *tentatively* scheduled for **Friday, September 23, 2022**, **Friday, October 28, 2022** and **Friday, December 02, 2022** — all of them during class time.

Make-up exams: Talk to me!

Final exam: It will happen during the finals week, as decided by the university.

Some important dates:

1. First day of classes: August 22, 2022.
2. Labor Day Holiday (no classes): Monday, September 05, 2022.
3. Automatic Grade of W for Dropped Course(s) for Undergraduate students: September 06–November 11, 2022
4. Last day to withdraw without petition to the Dean: November 13, 2022
5. Petition to College Dean to Drop Course(s) for Undergraduate students: November 14–December 09, 2022
6. Thanksgiving break (no classes): November 23-27, 2022
7. Final exam preparation period: December 04–December 11, 2022
8. Final exam period: December 12–16, 2022

For more important dates, please visit:
https://www.ou.edu/registrar/academic-calendars/spring-2022-academic-calendar

Policy on W/I grades: Through the end of the sixth week of the semester, students can withdraw from the course with an automatic W. Between the seventh and tenth weeks of the semester, undergraduate students can continue to withdraw with an automatic W, but graduate students must obtain the instructor's signature on the University's "drop form" to withdraw from the course, and along with the signature the instructor must indicate whether the student is passing or failing at the time of the withdrawal. After the tenth week of the semester, all students can only withdraw via petition to the Dean of their college. The petition process also requires the instructor's signature with a passing-failing indication at the time the petition is filed. Note that a "failing" indication on the petition means that even if the petition is approved the grade in the course will be weighted in your GPA as an F.

The grade of I is not intended to serve as a benign substitute for the grade of F, and is only given if a student has completed the majority of the work in the course at a passing level (for example everything except the final exam), the course work cannot be completed because of compelling and verifiable problem beyond the student's control, and the student expresses a clear intention of making up the missed work as soon as possible. Moreover, current University policies require that instructors and the affected students execute a written "Incomplete Contract" before a grade of I can be given. The contract makes clear: (1) what work is to be made up; (2) when the make-up work must be completed (which cannot be more than one calendar year from the assignment of the I); and (3) *what alternative grade will be assigned if the make-up work is not completed*. If the make-up work specified in the contract is not made up within one calendar year, then the alternative grade specified in the contract will be entered on the student's transcript. Thus the I grade does not became permanent on the transcript if it is not made up within one year.

Academic misconduct: Cheating is strictly prohibited at OU. It is the instructor's professional obligation to report academic misconduct to the Office of Academic Integrity Programs. Sanctions for academic misconduct can include expulsion from the University and an F in this course. Typically, all work on exams and quizzes must be yours and yours alone without any outside help. It is your responsibility to know and follow the rules set by the instructor. For more details on the University's policies concerning academic misconduct consult the link: https://www.ou.edu/integrity/students .

This link also has information about admonitions (essentially warnings about potential misconduct for fairly minor infractions) and your rights to appeal charges of academic misconduct.

Students with disabilities: Students requiring academic accommodation should contact the Accessibility and Disability Resource Center for assistance at (405)325-3852 or TDD: (405)325-4173. *If you have accommodations in place, please contact your instructor within the first week of classes (or within the first week of accommodations being approved for you) in order to discuss the relevant details for implementing your accommodations.* Note that depending on the specific accommodations requested and the structure of the course, you may need to be referred to the course coordinator to make appropriate arrangements for accommodations for uniform exams and/or quizzes. For more, please visit https://www.ou.edu/drc

Discrimination, Bias, Harassment: In light of incidents on other campuses and to further enhance responsiveness, the OU has established a 24-hour Reporting Hotline. The hotline will serve as an added protection for OU students, handling reports of bias, discrimination, physical or mental harassment or misconduct by OU community members. The 24-hour Reporting Hotline can be accessed by calling 844-428-6531 or going online to www.ou.ethicspoint.com. For more, visit https://www.ou.edu/eoo

Land Acknowledgment: Long before the University of Oklahoma was established, the land on which the University now resides was the traditional home of the "Hasinais" Caddo Nation and "Kirikir?i:s" Wichita & Affiliated Tribes. We acknowledge this territory once also served as a hunting ground, trade exchange point, and migration route for the Apache, Comanche, Kiowa and Osage nations. Today, 39 tribal nations dwell in the state of Oklahoma as a result of settler and colonial policies that were designed to assimilate Native people. The University of Oklahoma recognizes the historical connection our university has with its indigenous community. We acknowledge, honor and respect the diverse Indigenous peoples connected to this land. We fully recognize, support and advocate for the sovereign rights of all of Oklahoma's 39 tribal nations. This acknowledgment is aligned with our university's core value of creating a diverse and inclusive community. It is an institutional responsibility to recognize and acknowledge the people, culture and history that make up our entire OU Community.

Mental Health Support Services: If you are experiencing any mental health issues that are impacting your academic performance, counseling is available at the University Counseling Center located on the second floor of the Goddard Health Center at 620 Elm, Rm. 201. To schedule an appointment call (405)325-2911. For more, visit https://www.ou.edu/ucc

Pregnancy/Childbirth Accommodations: Should you need modifications or adjustments to your course requirements because of documented pregnancy-related or childbirth-related issues, please contact your professor as soon as possible to discuss. Generally, modifications will be made where medically necessary and similar in scope to accommodations based on temporary disability. For more, https://www.ou.edu/eoo/faqs/pregnancy-faqs

Recording: Class sessions of this course will typically be **not** recorded, with the exception of emergency situations. If and whenever they are, students may not share any course recordings with individuals not enrolled in the class nor are they to upload recordings to any other online environment.

Sexual Misconduct and Discrimination (Title IX resources): For any concerns regarding gender-based discrimination, sexual harassment, sexual assault, dating/domestic violence, or stalking, OU offers a variety of resources. To learn more or to report an incident, please contact the Sexual Misconduct Office at (405)325-2215 (8am-5pm, M-F) or smo@ou.edu. Incidents can also be reported confidentially to OU Advocates at (405)615-0013 (phones are answered 24 hours a day, 7 days a week). Be advised that a professor/GA/TA is required to report instances of sexual harassment, sexual assault, or discrimination to the Sexual Misconduct Office. Inquiries regarding non-discrimination policies can be directed to either a Title IX Coordinator or University Equal Opportunity Officer (405)325-3546 or smo@ou.edu. For more, see https://www.ou.edu/eoo.

DEPARTMENT OF MATHEMATICS

COURSE INFORMATION FOR MATH 2443–004 and MATH 2443–005

Calculus and Analytic Geometry IV

Fall 2021

Instructor: Dr. Hitesh Gakhar
E-mail: `hiteshgakhar@ou.edu`
Office: 1008 PHSC

Office Hours: **M** 4:00 PM–5:00 PM, **Th**: 2:00 PM–3:00 PM, and **by appointment**

WEEKDAYS	TIME	ITEM	LOCATION
Tuesday, Thursday	9:00 AM - 10:15 AM	2443–005 Class	PHSC 0321
Tuesday, Thursday	10:30 AM - 11:45 AM	2443–004 Class	PHSC 0115
Monday–Thursday	9:00 AM-5:00 PM	Math Center	
Friday	9:00 AM- 3:00 PM	Math Center	

For more about the Math Center, visit https://www.ou.edu/cas/mathcenter

Prerequisites: MATH 2433.

Text: *Calculus (8th ed)*, by James Stewart, Brooks/Cole, 2012. **Please note that even though my lecture notes will be loosely based on the book, the book itself is not necessary for the course. I will provide you with sufficient information.**

About the course: This course on calculus in more than one variables will be taught in a **flipped** setting. I will be recording and uploading a few videos before each lecture. You would be expected to diligently watch the videos before coming to class where we would spend our time working on problems together and fine-tuning the concepts. Some of these classes might start with a pop quiz based on the videos as a form of extra credit.

Expectations: I expect you to watch videos or read accompanying slides/notes before the class session. I also expect you to attend as many lectures as you can. I expect you to ask lots of questions. I expect you to frequently visit office hours or the Math Center. Above all, I expect you to be an engaged learner. By this I mean that you will engage your peers in and out of class, and ask plenty of questions of me.

Class Etiquette: Even though masking is not required by law, we should hold it as a community expectation of one another to keep us safe and together. In other words, I strongly request that you wear a mask in class.

Grading:

HOMEWORK	QUIZZES	EXAM 1	EXAM 2	EXAM 3	FINAL	TOTAL
20%	10%	15%	15%	15%	25%	100%

Homework: There will be problem sets each week either in WeBWork or written homeworks available on Canvas. *Tentatively*, there will be 12 written homeworks and the lowest 3 will be dropped.

Quizzes: There will be a quiz *almost* each week. *Tentatively*, there will be 12 quizzes, and the lowest 3 will be dropped.

Exams: Three exams that are are 75 minutes long and are *tentatively* scheduled for **Tuesday, September 21, 2021**, **Tuesday, October 26, 2021**, and **Thursday, December 02, 2021** — all of them during class time.

Make-up Policy: Talk to me!

Final exam: The final exam is during the final week on December 13, 2021 from 8:00AM–10:00AM (for Section 005), or December 16, 2021 from 8:00AM–10:00AM (for Section 004).

Some important dates:

1. First day of classes: Monday, August 23, 2021.
2. Labor Day Holiday (no classes): Monday, September 06, 2021.
3. Last day to withdraw with an automatic W: Friday, November 12, 2021 for undergraduate students
4. Last day to withdraw without petition to the Dean: Friday, November 14, 2021 (for graduate students a W/F grade is assigned for withdrawals processed during the period October 2–27).
5. Thanksgiving break (no classes): November 24-28, 2021.
6. Final exam preparation period: December 05–12, 2021
7. Final exam period: December 13–17, 2021 (consult the University's final exam schedule on `one.ou.edu` for the time period that corresponds to the meeting time for your course).

Policy on W/I grades:

Through the end of the sixth week of the semester, students can withdraw from the course with an automatic W. Between the seventh and tenth weeks of the semester, undergraduate students can continue to withdraw with an automatic W, but graduate students must obtain the instructor's signature on the University's "drop form" to withdraw from the course, and along with the signature the instructor must indicate whether the student is passing or failing at the time of the withdrawal. After the tenth week of the semester, all students can only withdraw via petition to the Dean of their college. The petition process also requires the instructor's signature with a passing-failing indication at the time the petition is filed. Note that a "failing" indication on the petition means that even if the petition is approved the grade in the course will be weighted in your GPA as an F.

The grade of I is not intended to serve as a benign substitute for the grade of F, and is only given if a student has completed the majority of the work in the course at a passing level (for example everything except the final exam), the course work cannot be completed because of compelling and verifiable problem beyond the student's control, and the student expresses a clear intention of making up the missed work as soon as possible. Moreover, current University policies require that instructors and the affected students execute a written "Incomplete Contract" before a grade of I can be given. The contract makes clear: (1) what work is to be made up; (2) when the make-up work must be completed (which cannot be more than one calendar year from the assignment of the I); and (3) *what alternative grade will be assigned if the make-up work is not completed*. If the make-up work specified in the contract is not made up within one calendar year, then the alternative grade specified in the contract will be entered on the student's transcript. Thus the I grade does not became permanent on the transcript if it is not made up within one year.

Academic misconduct: All cases of suspected academic misconduct will be reported to the Office of Academic Integrity Programs as possible violations of University's Academic Integrity Code. If the violation is confirmed by the Academic Integrity Program's Office, the penalties can be quite severe, so the best advice is **Don't do it!** For more details on the University's policies concerning academic misconduct consult the link

`http://integrity.ou.edu/students.html`

This link also has information about admonitions (essentially warnings about potential misconduct for fairly minor infractions) and your rights to appeal charges of academic misconduct.

Students are also bound by the provisions of the *OU Student Code*, available at

`https://www.ou.edu/content/dam/studentlife/documents/AllCampusStudentCode.pdf`

Students with disabilities: The University of Oklahoma is committed to providing reasonable accommodation for all students with disabilities. Students with disabilities who require accommodations in this course are requested to speak with the instructor as early in the semester as possible. Students with disabilities must be registered with the Office of Disability Services prior to receiving accommodations in this course. The Office of Disability Services (405–325–3852, `drc@ou.edu`) is in the University Community Center at 730 College Avenue, Norman, OK 73019.

Detailed Course Schedule

This schedule and the syllabus for individual quizzes/exams are tentative and subject to changes which will be announced in class.

Date:	Sections:	Summary
Week 1		
T 08/24	Introduction to MATH-2443	Welcome, Introductions, and Discussions
Th 08/26	14.1, 14.2	Functions of several variables, Limits and continuity
Week 2		
T 08/31	14.3	Partial derivatives
Th 09/02	14.4	Tangent planes and linear approximations
Week 3		
T 09/07	14.5	Chain rule
Th 09/09	14.6	Directional derivatives and gradients
Week 4		
T 09/14	14.7, 14.8	Maximum and minimum values, Lagrange multipliers
Th 09/16	**REVIEW**	Review for Exam 1
Week 5		
T 09/21	**EXAM 1**	Exam 1 (14.1–14.8)
Th 09/23	15.1	Double integrals over rectangles
Week 6		
T 09/28	15.2	Double integrals over general regions
Th 09/30	15.3, 15.4	Double integrals in polar coordinates, Applications
Week 7		
T 10/05	15.5	Surface Area
Th 10/07	15.6	Triple integrals
Week 8		
T 10/12	15.7	Triple integrals in cylindrical coordinates
Th 10/14	15.8	Triple integrals in spherical coordinates
Week 9		
T 10/19	15.9	Change of variables
Th 10/21	**REVIEW**	Review for Exam 2
Week 10		
T 10/26	**EXAM 2**	Exam 2 (15.1–15.9)
Th 10/28	16.1	Vector fields
Week 11		
T 11/02	16.2, 16.3	Line integrals, Fundamental theorem for line integrals
Th 11/04	16.4	Green's theorem
Week 12		
T 11/09	16.5	Curl and divergence
Th 11/11	16.6	Parametric surfaces and their areas
Week 13		
T 11/16	16.7	Surface integrals
Th 11/18	16.8	Stokes' theorem
Week 14		
T 11/23	16.9	The divergence theorem
Th 11/25	**NO CLASS**	Thanksgiving break
Week 15		
T 11/30	**REVIEW**	Review for Exam 3
Th 12/02	**EXAM 3**	Exam 3 (16.1–16.8)
Week 16		
T 12/07	**REVIEW**	Review for Final Exam
Th 12/09	**REVIEW**	Review for Final Exam

4. Curriculum: Worksheets and Assignments

In this section, I include samples of in-class worksheets (or problem lists) and homework assignments.

4.1. Linear Algebra I (MATH 3333, University of Oklahoma)

I have taught this course in *Fall 2022, Spring 2022, Spring 2021*. This course covers linear systems, Row Echelon forms, Matrices, Determinants, Inverses, Eigenvectors, Linear Transformations, Subspaces, Basis, Eigenbasis, General vector spaces like polynomial spaces, etc.

4.1.1 Worksheet on Eigenbasis and Diagonalization

The next two pages show Worksheet 09 from Fall 2022. This worksheet covers Eigenbasis & Diagonalization.

4.1.2 Homework Assignment on Subspaces and Spanning Sets

The two pages after Worksheet 09 show Homework 06 from Fall 2022. This assignment is based on the topic subspaces and spanning sets.

MATH 3333 **Worksheet 09 (Episodes 30-32)** **Fall 2022**

PLEASE DO NOT SUBMIT THIS IN PLACE OF HOMEWORK!

Solve all the problems before looking up the solutions. You work must be detailed and neat.

Timeline: Week 11 (October 30–November 05, 2022)

References: Lecture/Episode 30: Eigenbases, Lecture/Episode 31: Finding Eigenbases, Lecture/Episode 32: Diagonalization

1 Is this an eigenbasis?

Do the vectors of S form an eigenbasis for A?

$$S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{1}{3} \\ 1 \end{bmatrix} \right\} \qquad A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & -1 \\ 1 & 3 & -2 \end{bmatrix}$$

2 Matrix Multiplication using eigenbasis

Compute $A^{10}\mathbf{v}$ using the basis S if

$$S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{1}{3} \\ 1 \end{bmatrix} \right\} \qquad A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & -1 \\ 1 & 3 & -2 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Hint: Use your knowledge from Exercise 1.

3 Does this matrix have an eigenbasis?

You don't need to compute an eigenbasis.

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 0 & 6 \\ 0 & -3 & 0 \\ 5 & 0 & 2 \end{bmatrix}$

4 Find eigenbasis!

Find an eigenbasis or explain why it cannot exist!

(a) $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 2 \\ 2 & 3 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 7 & 0 & -4 \\ 0 & 5 & 0 \\ 5 & 0 & -2 \end{bmatrix}$

5 Geometric multiplicity vs Algebraic Multiplicity

Show that the geometric multiplicity is smaller than algebraic multiplicity for an eigenvalue of $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$.

6 Diagonalize this matrix!

Diagonalize these matrices if possible!

(a) $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 2 \\ 2 & 3 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 7 & 0 & -4 \\ 0 & 5 & 0 \\ 5 & 0 & -2 \end{bmatrix}$

7 Orthogonality

Is $S = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$ an orthogonal eigenbasis of $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$?

8 Is this diagonalizable?

Is $\begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 6 & 7 & 2 \\ 5 & 7 & 1 & 4 \\ 7 & 2 & 4 & 0 \end{bmatrix}$ diagonalizable?

Suggested exercises for more practice!

Section 3.3 of the book: Exercises 3.3.1 , **Section 5.5** of the book: Exercises 5.5.4, (see Canvas modules for the book and the solution manual)

MATH 3333 **Homework 06** **Due at 10/10/2022, 11:59 PM**

Name: ______________________________ Section : __________

INSTRUCTIONS: Present your solution to each problem in a clear and orderly fashion. You must show your work. An answer alone without supporting work **will not** receive full credit. Upload the solutions to Gradescope before the deadline.

ACKNOWLEDGMENTS: What resources did you use? Who would you like to thank? Did you do this entirely by yourself?

TOTAL POINTS = 30 **TOTAL PROBLEMS = 5**

1. (4 points) **Short Answer Questions**! Reasoning welcomed, but not needed for this problem.
   (a) (1 point) What is the kernel of a matrix A? Be precise.
   (b) (1 point) What is the image space of a matrix A? Be precise.
   (c) (1 point) S is a spanning set for V if every element of V can be written as a linear combination in ____________ way(s). What should it be? At most one? At least one? Exactly one? Zero? Infinitely many? Two? Three? Four? or Something else?
   (d) (1 point) (T/F) Even though the $n \times 1$ zero vector is not considered an eigenvector, it is still a part of λ-eigenspace for the $n \times n$ matrix A.

2. (5 points) Show that $V = \left\{ \begin{bmatrix} 5s \\ s^2 \\ t+2 \\ r \end{bmatrix} \middle| r, s, t \in \mathbb{R} \right\}$ is **not** a subspace of $\mathbb{R}^4$.
   Expectations: I want you to write each step and reasoning, and have a proper conclusion. See classwork for formatting.

3. (7 points) Show that $V = \left\{ \begin{bmatrix} 2s+3t \\ s \\ t \end{bmatrix} \middle| s, t \in \mathbb{R} \right\}$ is a subspace of $\mathbb{R}^3$.
   Expectations: I want you to start with arbitrary vectors, write each step and reasoning, and have a proper conclusion. See classwork for formatting.

4. (7 points) Show that $\left\{ \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ 3 \\ 13 \end{bmatrix} \right\}$ is **not** a spanning set of $\mathbb{R}^3$.
   Expectations: I want you to write each step and reasoning, and have a proper conclusion. See classwork for formatting.

5. (7 points) Show that $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \right\}$ is a spanning set of the following subspace of $\mathbb{R}^4$:

$$U = \left\{ \begin{bmatrix} r \\ r+s \\ s \\ 0 \end{bmatrix} \middle| r, s \in \mathbb{R} \right\}$$

Expectations: I want you to start with arbitrary vectors, write each step and reasoning, and have a proper conclusion. See classwork for formatting.

4.2. Calculus & Analytic Geometry (MATH 2443, University of Oklahoma)

I have taught this course in *Fall 2021, Fall 2020.* This course covers multivariable differentiation, tangent planes, chain rule, gradient, maximum and minimum, double integration, polar integration, cylindrical coordinates, spherical coordinates, divergence, curl, Green's theorem, Stoke's theorem, Divergence theorem, etc.

4.2.1 Sample Lesson Plan on Partial Derivatives

Course: Calculus & Analytic Geometry IV, **Format**: Flipped Course

- **Last Time:** Limits and Continuity for Multivariable Functions
- **TODO:** Solicit any Questions, Comments, Queries, and Concerns.
- **TODO:** Spend five minutes reviewing the material from the videos:
  - Review how multivariable functions demand a notion rate of change in each direction, specifically parallel to the axes. Briefly comment that we will later learn to do this in arbitrary directions.
  - Review the definition of partial derivatives and re-explain why while differentiating with respect to one of the variables, the other variables are treated as a constant.
- **TODO**: Solve the following problem in the interactive style with the whole class:
  - If $p(x,y) = x^4 + 3x^2y + xy^2 + y^3$, find the partial derivatives $\frac{\partial p}{\partial x}$ and $\frac{\partial p}{\partial y}$.
- **TODO**: Briefly review-by-soliciting product rule, quotient rule, chain rule from single variable calculus.
- **TODO**: Assign any 2 out of the following problems, give them 5-10 minutes and discuss them in the interactive style with the whole class:
  - If $f(x,y) = y\ln(x^2 + x + 5)$, find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.
  - If $g(x,y) = e^{x+y^3}$, find the partial derivatives $\frac{\partial g}{\partial x}$ and $\frac{\partial g}{\partial y}$.
  - If $h(x,y) = \sin(5x^2 + y)$, find the partial derivatives $\frac{\partial h}{\partial x}$ and $\frac{\partial h}{\partial y}$.
- **TODO**: Briefly review what second order partial derivatives are. Assign any 2 out of the following problems, give them 5-10 minutes and discuss them in the interactive style with the whole class:
  - If $z = \frac{x}{\sqrt{x^2+y^2}}$, find the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
  - If $r(x,y) = \cos\left(\frac{x}{y}\right)$, find the partial derivatives $\frac{\partial^2 r}{\partial x^2}$, $\frac{\partial^2 r}{\partial x \partial y}$, $\frac{\partial^2 r}{\partial y \partial x}$ and $\frac{\partial^2 r}{\partial y^2}$.
- **TODO**: Mention that the whole process can be extended to more than 2 variables. Assign the following problem, give them 5 minutes and discuss them in the interactive style with the whole class:
  - If $w(x,y,z) = xyz$, find the partial derivatives $\frac{\partial w}{\partial x}$, $\frac{\partial w}{\partial y}$, and $\frac{\partial w}{\partial z}$.
- **TODO**: Use the following problem to motivate the need for implicit differentiation. Solve the problem in the interactive style with the whole class:
  - If $x^2 + 2y^2 + 3z^2 = 1$, find the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial w}{\partial y}$. **Hint: Implicit Differentiation.**
- **Next Time**: Tangent Planes

4.2.2 Worksheet on Chain Rule, Directional Derivatives, and Gradient

The next page shows Worksheet 03 from Fall 2021. This worksheet covers Chain Rules, Directional Derivatives, Gradient, and Tangent Planes to Level Surfaces.

4.2.3 Homework on Double Integrals and Polar Integrals

The five pages after Worksheet 03 show Homework 06 from Fall 2021. This assignment is based on the topic of double integrals and polar integrals. The assignment is accompanied by the solutions.

MATH 2443 Worksheet: Week 3 (Sections 14.5-14.6) Fall 2021

Solve all the problems before looking up the solutions. You work must be detailed and neat.

References: 14.5: Chain Rule, 14.6: Directional Derivatives and Gradient Vector

1 Chain Rule

1. Find $\frac{dz}{dt}$ if $z = xy^3 - x^2y$ where $x = t^2 + 1$ and $y = t^2 - 1$.
2. Find $\frac{dw}{dt}$ if $w = \ln(\sqrt{x^2 + y^2 + z^2})$ where $x = \sin(t)$, $y = \cos(t)$, and $z = \tan(t)$.
3. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ if $z = \ln(3x + 2y)$ where $x = s\sin(t)$, and $y = t\cos(s)$.
4. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ if $z = e^r\cos(\theta)$ where $r = st$, and $\theta = \sqrt{s^2 + t^2}$

2 Directional Derivatives

1. If $f(x, y) = \frac{x}{x^2+y^2}$, find the directional derivative of f at $(0, \pi/3)$ in the direction of $\vec{u} = \langle -6, 8\rangle$.
2. If $f(x, y) = y\cos(xy)$, find the directional derivative of f at $(0, 1)$ in the direction of $\vec{u} = \langle 1, 1\rangle$.
3. If $f(x, y, z) = x\sin(yz)$, find the directional derivative of f at $(1, 3, 0)$ in the direction of $\vec{u} = \langle 1, 2, -1\rangle$.

3 The Gradient Vector and Maximal Rate of Change

1. If $g(x, y) = \sin(x) + e^{xy}$, find the gradient ∇g.
2. If $h(x, y, z) = xy^2z - xyz^3$, find the gradient ∇h.
3. If $f(s, t) = te^{st}$, find the maximum rate of change of f at $(0, 2)$ and the direction in which it occurs.
4. If $f(x, y) = \sin(xy)$, find the maximum rate of change of f at $(1, 0)$ and the direction in which it occurs.

4 Tangent Planes to Level Surfaces

1. Find the equation of the tangent plane and the normal line to the level surface $xy^2z^3 = 8$ at $(2, 2, 1)$.
2. Find the equation of the tangent plane and the normal line to the level surface $x^4 + y^4 + z^4 = 3x^2y^2z^2$ at $(1, 1, 1)$.

   .

MATH 2443 **Homework 6** **Due at 10/05/2021, 11:59 PM**

Name: ______________________________ Section : __________

INSTRUCTIONS: Present your solution to each problem in a clear and orderly fashion. You must show your work. An answer alone without supporting work **will not** receive full credit. Upload the solutions to Gradescope before the deadline.

TOTAL POINTS = 20 **TOTAL PROBLEMS = 4 + 1**

1. (5 points) Evaluate $\iint\limits_D y\sqrt{x^2-y^2}\, dA$ where $D = \{(x,y) \mid 0 \le y \le x,\ 0 \le x \le 2\}$.

Solution: We first write this double integral in the iterated integral form:

$$\iint\limits_D y\sqrt{x^2-y^2}\, dA = \int\limits_0^2 \int\limits_0^x y\sqrt{x^2-y^2}\, dydx$$

Then we integrate with respect to y first and then with respect to x. To begin, let $u = x^2 - y^2$ which implies $du = -2ydy$ and since the limits of integration for y are 0 and x, the limits on integration for u are x^2 and 0:

$$\int\limits_0^2 \int\limits_0^x y\sqrt{x^2-y^2}\, dydx = \int\limits_0^2 \int\limits_{x^2}^0 \frac{-1}{2}\sqrt{u}\, dudx = \int\limits_0^2 \left[\frac{-1}{2\frac{3}{2}} u^{3/2}\right]_{x^2}^0 \, dx$$

$$= \int\limits_0^2 \frac{-1}{3}(-x^3)\, dx = \left[\frac{x^4}{12}\right]_0^2 = \frac{2^4}{12} = \frac{4}{3}$$

Therefore,

$$\iint\limits_D y\sqrt{x^2-y^2}\, dA = \frac{4}{3}$$

2. (5 points) Find the area of the region D inside the circle $(x-1)^2 + y^2 = 1$ and outside the circle $x^2 + y^2 = 1$.

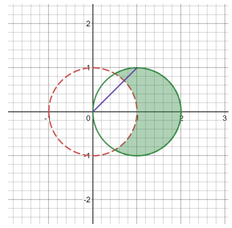

Figure 1: From Question 2. The region D.

Solution: To find the area of D, we must first understand it. We start by finding the points of intersection of the two circles $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$:

$$\begin{aligned} &1-(x-1)^2 = y^2 = 1 - x^2 \implies 1 - x^2 - 1 + 2x = 1 - x^2 \implies x = \frac{1}{2} \\ &\implies y^2 = 1 - \left(\frac{1}{2}\right)^2 \implies y = \pm\frac{\sqrt{3}}{2} \end{aligned}$$

Therefore, $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ are the points of intersection. See Figure [ref] for more details. In polar coordinates, these points correspond to $\left(1, \frac{\pi}{3}\right)$ and $\left(1, -\frac{\pi}{3}\right)$ because

$$r^2 = \left(\frac{1}{2}\right)^2 + \left(\pm\frac{\sqrt{3}}{2}\right)^2 = 1 \text{ and } \tan(\theta) = \frac{y}{x} = \pm\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \pm\sqrt{3}$$

We also need to write the two equations of the circles in polar coordinates using $x = r\cos(\theta)$ and $y = r\sin(\theta)$:

$$\begin{aligned} &(x-1)^2 + y^2 = 1 \Rightarrow x^2 + 1^2 - 2x + y^2 = 1 \\ \Rightarrow &(r\cos(\theta))^2 + 1^2 - 2(r\cos(\theta)) + (r\sin(\theta))^2 = 1 \Rightarrow r = 2\cos(\theta) \end{aligned}$$

and

$$x^2 + y^2 = 1 \Rightarrow (r\cos(\theta))^2 + (r\sin(\theta))^2 = 1 \Rightarrow r = 1$$

With this we can write our domain:

$$D = \left\{(r,\theta) \mid -\frac{\pi}{3} \leq \theta \leq \frac{\pi}{3}, 1 \leq r \leq 2\cos(\theta)\right\}$$

Then the area computation becomes:

$$\begin{aligned} \iint_D 1\, dA &= \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \int_1^{2\cos(\theta)} 1\, r dr d\theta = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left[\frac{r^2}{2}\right]_1^{2\cos(\theta)} d\theta = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left(2\cos^2(\theta) - \frac{1}{2}\right) d\theta \\ &= \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left(1 + \cos(2\theta) - \frac{1}{2}\right) d\theta = \left[\theta + \frac{\sin(2\theta)}{2} - \frac{\theta}{2}\right]_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \\ &= \left(\frac{\pi}{3} + \frac{\sin\left(2\frac{\pi}{3}\right)}{2} - \frac{\pi}{6}\right) - \left(-\frac{\pi}{3} + \frac{\sin\left(-2\frac{\pi}{3}\right)}{2} - \frac{-\pi}{6}\right) \\ &= 2\left(\frac{\pi}{3} + \frac{\sin\left(2\frac{\pi}{3}\right)}{2} - \frac{\pi}{6}\right) = \frac{\pi}{3} + \frac{\sqrt{3}}{2} \end{aligned}$$

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Therefore, the area of D is $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$.

3. (5 points) Change to polar coordinates and evaluate: $\iint\limits_D e^{-x^2-y^2}\, dA$ where D is the region bounded by the semi-circle $y = \sqrt{16 - x^2}$ and the x-axis.

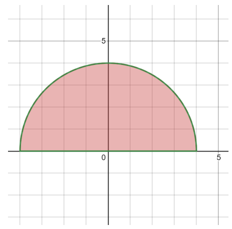

Figure 2: From Question 3. The region D.

Solution: To convert into polar coordinates, we need to convert the integrand and write D in polar coordinates. First, the integrand can be converted as:

$$e^{-x^2-y^2} = e^{-(x^2+y^2)} = e^{-r^2}$$

$$D = \{(r, \theta) \mid 0 \le r \le 4\ ,\ 0 \le \theta \le \pi\}$$

Now, we can convert the integral and integrate:

$$\iint\limits_D e^{-x^2-y^2}\, dA = \int_0^{\pi}\int_0^{4} e^{-r^2}\, r dr d\theta$$

$$= \int_0^{\pi}\int_0^{16} \frac{e^{-u}}{2}\, du d\theta \quad (\text{ Substituted } u = r^2,\ du = 2rdr)$$

$$= \int_0^{\pi} \left[\frac{e^{-u}}{-2}\right]_0^{16} d\theta = \int_0^{\pi} \left(\frac{1}{2} - \frac{e^{-16}}{2}\right) d\theta$$

$$= \left[\left(\frac{1}{2} - \frac{e^{-16}}{2}\right)\theta\right]_0^{\pi} = \left(\frac{1}{2} - \frac{e^{-16}}{2}\right)\pi$$

Therefore,

$$\iint\limits_D e^{-x^2-y^2}\, dA = \left(\frac{1}{2} - \frac{e^{-16}}{2}\right)\pi$$

4. (5 points) Use polar coordinates to find the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 16$ and outside the cylinder $x^2 + y^2 = 4$.

Page 3

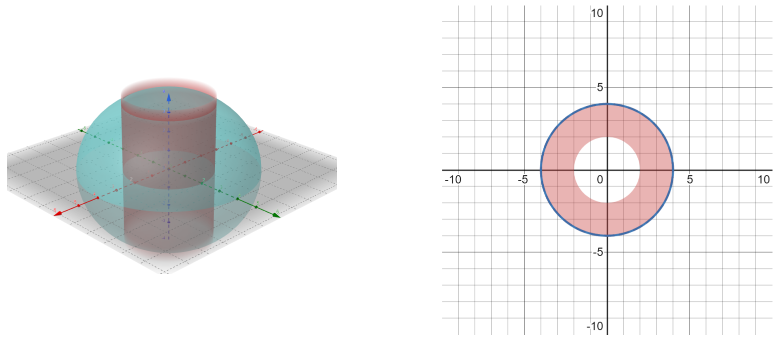

Figure 3: From Question 4. Left: The 3D solid. Right: Its projection into xy-plane.

Solution: Instead of finding the volume of S directly, we will find the volume of its top half H and then multiply by 2. We can do so because the shape is symmetric about the xy-plane. The region H that lies inside the sphere $x^2 + y^2 + z^2 = 16$ and outside the cylinder $x^2 + y^2 = 4$ can be viewed as the region under the surface of the top hemisphere $z = \sqrt{16 - x^2 - y^2}$ over the domain equal to an annulus of inner radius 2 and outer radius 4. That is, D can be written as:

$$D = \{(r, \theta) \mid 2 \leq r \leq 4\ ,\ 0 \leq \theta \leq 2\pi\}$$

With this in hand, we can compute the integral:

$$\begin{aligned}
Vol(H) &= \iint\limits_D z\, dA = \int_0^{2\pi}\int_2^4 \sqrt{16 - r^2}\, r dr d\theta \\
&= \int_0^{2\pi}\int_{12}^{0} \frac{\sqrt{u}}{-2}\, du d\theta \quad (\text{ Substituted } u = 16 - r^2,\ du = -2rdr) \\
&= \int_0^{2\pi} \left[\frac{(u)^{\frac{3}{2}}}{-2\frac{3}{2}}\right]_{12}^{0} d\theta = \int_0^{2\pi} \left(0 - \frac{(12)^{\frac{3}{2}}}{-2\frac{3}{2}}\right) d\theta \\
&= \frac{(12)^{\frac{3}{2}}}{3}\,[\theta]_0^{2\pi} = \frac{(12)^{\frac{3}{2}}}{3} 2\pi = 12\frac{(12)^{\frac{1}{2}}}{3} 2\pi
\end{aligned}$$

Therefore, the volume of the whole solid S is:

$$Vol(S) = 2Vol(H) = 2\left(12\frac{(12)^{\frac{1}{2}}}{3}2\pi\right) = 16(3)^{\frac{1}{2}}2\pi$$

5. (0 points) **CHALLENGE PROBLEM**: Integrate

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dydx$$

Solution: We will solve this via polar coordinates. The integrand becomes $\sqrt{x^2+y^2} = r$, while the differentials $dydx$ become $rdrd\theta$. As for the limits, we first need to identify the domain of integration. We know that $0 \leq x \leq 2$, and for each x, $0 \leq y \leq \sqrt{2x-x^2}$. The right hand inequality can be rewritten as

$$y \leq \sqrt{2x-x^2} \Rightarrow y^2 \leq 2x - x^2 \Rightarrow y^2 + x^2 - 2x + 1 \leq 1 \Rightarrow (x-1)^2 + y^2 \leq 1$$

that is the unit disk centered at $(1,0)$. The boundary circle $(x-1)^2 + y^2 = 1$ can be converted to polar:

$$(x-1)^2 + y^2 = 1 \mapsto r = 2\cos(\theta)$$

Hence, we can write the domain D in polar coordinates:

$$D = \left\{(r,\theta) \mid 0 \leq r \leq 2\cos(\theta)\ ,\ 0 \leq \theta \leq \frac{\pi}{2}\right\}$$

So now we convert the integral and integrate:

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dydx = \int_0^{\frac{\pi}{2}} \int_0^{2\cos(\theta)} r\ rdrd\theta = \int_0^{\frac{\pi}{2}} \left[\frac{r^3}{3}\right]_0^{2\cos(\theta)} d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{(2\cos(\theta))^3}{3}\, d\theta = \frac{8}{3}\int_0^{\frac{\pi}{2}} (\cos(\theta) - \cos(\theta)\sin^2(\theta))\, d\theta = \frac{8}{3}\left[\sin(\theta) - \frac{\sin^3(\theta)}{3}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{8}{3}\left(\sin\left(\frac{\pi}{2}\right) - \frac{\sin^3\left(\frac{\pi}{2}\right)}{3} - \sin(0) + \frac{\sin^3(0)}{3}\right) = \frac{8}{3}\left(1 - \frac{1}{3}\right) = \frac{16}{9}$$

Therefore,

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dydx = \frac{16}{9}$$

4.3. Transitions to Proofs (MTH 299, Michigan State University)

I have taught this course in *Fall 2019, Fall 2018, Summer 2018, Spring 2017*. This course serves as an introduction and transition to proof-writing in Mathematics. The students enrolled in this course were mainly mathematics majors and minors, and they take it before they take advanced mathematics courses. The course content includes propositional logic, elementary set theory, proof techniques such as contradiction and induction, and some applications of the aforementioned in real analysis and number theory.

4.3.1 Homework on Convergence and Number Theory

The next three pages show Homework 09 from Fall 2019. This assignment is based on the topics of convergence of sequences and modular arithmetic from elementary number theory. The assignment is accompanied by the solutions.

4.3.2 Short Answer Practice Problems

The eight pages after Homework 09 show a Practice Worksheet from Fall 2019. This review was prepared for the pre-finals week to help prepare students for the exam.

Homework 9; Due Monday, 11/04/2019

Question 1. For each sequence $\{a_n\}_{n\in\mathbb{N}}$ say if it is convergent or not. If it is convergent, state its limit. No proof is necessary.

(a) $a_n = \dfrac{n^2}{n^3-2}$

(b) $a_n = \dfrac{n^2-n+1}{n(n+1)}$

(c) $a_n = \log(n)$

(d) $a_n = \sin\left(\dfrac{(2n-1)\pi}{2}\right)$

(e) $a_n = \dfrac{3^{n+1}+1}{3^n+2}$

Solution. (a) The sequence is convergent to 0.

(b) The sequence is convergent to 1.

(c) The sequence is divergent.

(d) The sequence is divergent because the series alternates between 1 and -1.

(e) The sequence is convergent to 3.

Question 2. Determine whether the following statements are **TRUE** or **FALSE**.

(a) $\forall n \in \mathbb{N}$, $n \mid 0$.

(b) Let $a, b \in \mathbb{Z}$. If $6 \nmid b$ then $6 \nmid ab$.

(c) $\forall n \in \mathbb{Z} \setminus \{0\}$, $n \equiv 0 \mod n$.

(d) Let $n \in \mathbb{Z}$. Then $n \equiv 7 \mod 2$ if and only if n is odd.

Solution. (a) TRUE. For each $n \in \mathbb{N}$, one can write $0 = (n)(0)$.

(b) FALSE. For example, if $a = 2$, $b = 3$, then $6 \nmid b$, but $6|ab$.

(c) TRUE. $\forall n \in \mathbb{Z} \setminus \{0\}$, $n - 0 = (1)n$, and since $1 \in \mathbb{Z}$, we obtain $n \equiv 0 \mod n$.

(d) TRUE. Observe that $n \equiv 7 \mod 2$ if and only if $n - 7 \equiv 2k$ for some k, which is equivalent to saying $n = 2k + 7 = 2(k+3) + 1$.

Question 3. Consider the sequence $\{a_n\}_{n\in\mathbb{N}}$ where $a_n = 1 - \dfrac{2\cos(n)}{n}$. Is the sequence convergent or not? If it is convergent, then prove your answer.

Solution. SCRATCH WORK: As $n \to \infty$, $\frac{2\cos(n)}{n} \to 0$ becomes really small, and consequently, $L = 1$ is a good candidate for the limit of the sequence a_n. We want $\left|1 - \dfrac{2\cos(n)}{n} - 1\right| < \epsilon$. Let us work backwards:

$$\left|1 - \frac{2\cos(n)}{n} - 1\right| < \epsilon \Leftrightarrow \left|\frac{2\cos(n)}{n}\right| < \epsilon$$

Manipulating the term on the right is hard because of how $\cos(n)$ behaves, however, there is an intermediate term that we can use:

$$\left|\frac{2\cos(n)}{n}\right| \leq \frac{2}{n} < \epsilon.$$

That's the same as requiring $n > \frac{2}{\epsilon}$.
PROOF: We claim that the given sequence converges to 1 and we will prove it now. Let $\epsilon > 0$ be generic. Since $\epsilon \neq 0$, then $\frac{2}{\epsilon} \in \mathbb{R}$. Then by the Archimedean Property, $\exists N \in \mathbb{N}$, satisfying $N > \frac{2}{\epsilon}$. Recall that $|\cos(n)| \leq 1$ for all n. Now, observe that for all $n \geq N$,

$$|a_n - L| = \left|1 - \frac{2\cos(n)}{n} - 1\right| = \left|\frac{2\cos(n)}{n}\right| = \frac{2|\cos(n)|}{n} \leq \frac{2}{n} \leq \frac{2}{N} < \frac{2}{2/\epsilon} = \epsilon.$$

Therefore, $\forall \epsilon > 0$, $\exists N \in \mathbb{N}$, such that $|a_n - 1| < \epsilon$. This finishes the proof.

Question 4. Prove that $\displaystyle\lim_{n\to\infty} \frac{3n+1}{4n+1} = \frac{3}{4}$.

Solution. SCRATCH WORK: We require $\left|\dfrac{3n+1}{4n+1} - \dfrac{3}{4}\right| < \epsilon$, which is equivalent to requiring

$$\left|\frac{12n+4-12n-3}{(4n+1)4}\right| < \epsilon \Leftrightarrow \left|\frac{1}{16n+4}\right| < \epsilon \Leftrightarrow \frac{1}{\epsilon} < 16n+4 \Leftrightarrow \frac{1}{16}\left(\frac{1}{\epsilon} - 4\right) < n.$$

PROOF. Let $\epsilon > 0$ be generic. Since $\epsilon \neq 0$, then $\frac{1}{16}\left(\frac{1}{\epsilon} - 4\right) \in \mathbb{R}$. Then by the Archimedean Property, $\exists N \in \mathbb{N}$, satisfying $N > \frac{1}{16}\left(\frac{1}{\epsilon} - 4\right)$. This is equivalent to saying that $16N > \frac{1}{\epsilon} - 4$, which is further equivalent to $\dfrac{1}{16N+4} < \epsilon$. Observe that for all $n \geq N$,

$$\left|\frac{3n+1}{4n+1} - \frac{3}{4}\right| = \left|\frac{12n+4-12n-3}{(4n+1)4}\right| = \frac{1}{16n+4} \leq \frac{1}{16N+4} < \epsilon,$$

where the second last step comes from $n \geq N \Rightarrow 16n+4 \geq 16N+4$ and the last step comes from discussion above. Therefore, $\forall \epsilon > 0$, $\exists N \in \mathbb{N}$, such that $\left|\dfrac{3n+1}{4n+1} - \dfrac{3}{4}\right| < \epsilon$. This finishes the proof.

Question 5. Assume that $a, b \in \mathbb{Z}$. Prove that $(a+b)^3 \equiv a^3 + b^3 \mod 3$.
This statement is sometimes called "Freshman's dream" since $(a+b)^3 = a^3 + b^3$ is a common algebraic mistake that freshman, and many others, make. This statement is true, however, when cosidered modulo 3.

Solution. Recall that $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$. We have to show that $(a+b)^3 - (a^3+b^3)$ is divisible by 3. Observe that

$$(a+b)^3 - (a^3+b^3) = a^3 + 3a^2b + 3ab^2 + b^3 - (a^3+b^3) = 3a^2b + 3ab^2 = 3(a^2b + ab^2)$$

where $(a^2b + ab^2) \in \mathbb{Z}$ since $a, b \in \mathbb{Z}$. Therefore, $(a+b)^3 - (a^3+b^3)$ is divisible by 3, or equivalently $(a+b)^3 \equiv (a^3+b^3) \mod 3$.

Short Answer Practice Problems: MTH299-006

0. Provide examples of the sequences described below.
   (a) A convergent sequence $\{a_n\}_{n\in\mathbb{N}}$ such that $a_n < a_{n+1}$.

   (b) A convergent sequence $\{a_n\}_{n\in\mathbb{N}}$ such that $a_{n+1} < a_n$.

   (c) A bounded sequence that is not convergent.

0. Provide examples of the functions described below.
   (a) A function, $f : \mathbb{N} \to \mathbb{N}$ such that f is injective but not surjective.

   (b) A function, $f : \mathbb{R} \to \mathbb{R}$ such that f is injective but not surjective.

   (c) A function, $f : \mathbb{N} \to \mathbb{N}$ such that f is surjective but not injective.

   (d) A function, $f : \mathbb{R} \to [0, \infty)$ such that f is surjective but not injective.

   (e) A function, $f : \mathbb{R} \to \mathbb{R}$ such that f is surjective but not injective.

0. Find the following indexed intersections and unions.

(a) Given $B_n = (n, n+1]$, find $\bigcap_{n=1}^{\infty} B_n$ and $\bigcup_{n=0}^{\infty} B_n$.

$\bigcap_{n=1}^{\infty} B_n =$ ______________, $\bigcup_{n=0}^{\infty} B_n =$ ______________

(b) Given $B_n = [0, n+1)$, find $\bigcap_{n=1}^{\infty} B_n$ and $\bigcup_{n=0}^{\infty} B_n$.

$\bigcap_{n=1}^{\infty} B_n =$ ______________, $\bigcup_{n=0}^{\infty} B_n =$ ______________

(c) Given $A_n = \left[-3 + \frac{2}{n}, 2 - \frac{1}{n}\right)$, find $\bigcap_{n=1}^{\infty} A_n$ and $\bigcup_{n=0}^{\infty} A_n$.

$\bigcap_{n=1}^{\infty} A_n =$ ______________, $\bigcup_{n=0}^{\infty} A_n =$ ______________

(d) Given $A_n = \left(-3 + \frac{2}{n}, 2 - \frac{1}{n}\right]$, find $\bigcap_{n=1}^{\infty} A_n$ and $\bigcup_{n=0}^{\infty} A_n$.

$\bigcap_{n=1}^{\infty} A_n =$ ______________, $\bigcup_{n=0}^{\infty} A_n =$ ______________

(e) Given $A_n = \left[1 - \frac{2}{n}, 2 + \frac{2}{n}\right)$, find $\bigcap_{n=1}^{\infty} A_n$ and $\bigcup_{n=0}^{\infty} A_n$.

. $\bigcap_{n=1}^{\infty} A_n =$ ______________, $\bigcup_{n=0}^{\infty} A_n =$ ______________

(f) Given $A_n = \left(1 - \frac{2}{n}, 2 + \frac{2}{n}\right]$, find $\bigcap_{n=1}^{\infty} A_n$ and $\bigcup_{n=0}^{\infty} A_n$.

$\bigcap_{n=1}^{\infty} A_n =$ ______________, $\bigcup_{n=0}^{\infty} A_n =$ ______________

0. Find the partitions of the given set satisfying the given properties.
   (a) Given that $X = \{A, B\}$ is a partition of $\{1, 2, 3\}$ find A and B. (*There could be more than one possible answer.*)

   (b) Let $X = \{A, B, C\}$ a partition of $\mathbb{Z}$. Suppose that $\forall a \in A$ we have $a \leq -3$ and $\forall b \in B$ we have $b \geq -3$. Give an example of A, B and C that satisfies above properties. (*There could be more than one such partition*).

   (c) Let $X = \{A, B, C\}$ be a partition of $[0, \infty)$. Given that $B = (2, \infty)$ and $A \cap \{x \in \mathbb{R} : x > 0\} = \emptyset$, find A and C.

0. Give examples of **three different integers** a, b, c such that;

(a) $\gcd(a,b) = \gcd(b,c) = \gcd(a,c)$.

$a =$ ____________, $b =$ ____________, $c =$ ____________

(b) $\gcd(a,b) = \gcd(b,c)$ but $\gcd(a,c) \neq \gcd(b,c)$.

$a =$ ____________, $b =$ ____________, $c =$ ____________

(c) All of $\gcd(a,b)$, $\gcd(b,c)$ and $\gcd(a,c)$ are different.

$a =$ ____________, $b =$ ____________, $c =$ ____________

0. Negate the following statements and write the truth value of the original statement.

(a) $\forall x \in \mathbb{R} \setminus \mathbb{Q}$, $x > \sqrt{2}$ or $x < \sqrt{2}$.

(b) $\forall p_1, p_2 \in \mathbb{P}_3$, $\big(p_1'(0) = p_2'(0) \Rightarrow p_1 = p_2\big)$

(c) $\exists x \in \mathbb{R}$ such that $\{x\} \cap (\infty, 0) = \emptyset$ and $\{x\} \cap (-\infty, 0) = \emptyset$

(d) $\exists y \in \mathbb{Z}$ such that $\forall x \in \mathbb{Z}$, $x < y$

0. Given the value of a_n (for $n \in \mathbb{N}$) find whether or not $\lim_{n\to\infty} a_n$ exists. If the limit exists find its value.

(a) $a_n = \dfrac{27n^2 + 2n - 3}{3n^3 + 2n^2 + n + 11}$

(b) $a_n = \dfrac{2n^3}{2n^3 + 5}$

(c) $a_n = \begin{cases} \dfrac{-n}{n^2+1}; & \text{if } n \text{ is odd} \\ \dfrac{n}{n^2+1}; & \text{if } n \text{ is even} \end{cases}$

(d) $a_n = \dfrac{\sin(n)\cos(n^2+1)}{n^2 + 2n + 1}$

0. Describe the distinct equivalence classes of the given equivalence relation
   (a) Equivalence relation on $\mathbb{Z}$ given by $n \sim m$ is and only if $2|(n-m)$.

   (b) Let $f(x) = x^2 - 1$ then let $\sim$ be the equivalence relation on $\mathbb{R}$ such that $a \sim b$ if and only if $f(a) = f(b)$.

(c) Equivalence relation on $\mathbb{R}^2$ given by $(x_1, y_1) \sim (x_2, y_2)$ if and only if $x_1^2 + y_1^2 = x_2^2 + y_2^2$.

0. Are the following sets bounded or unbounded? If bounded, give a bound.

(a) $A = \{3z + 1 : z \in \mathbb{N}\}$

(b) $B = \{\sin(n) : n \in \mathbb{R}\}$

(c) $C = \left\{\frac{r^{2k}}{r^{2k}+1} : r \in \mathbb{R}\right\}$ where k is any natural number.

(d) $D = \left\{\frac{r^{2k-1}}{r^{2k-1}+1} : r \in \mathbb{R}\right\}$ where k is any natural number.

0. Give examples of:
   (a) a relation on $\mathbb{R}$ that is transitive but not symmetric or reflexive.

   (b) a relation on $\mathbb{R} \times \mathbb{R}$, that is reflexive, symmetric, but not transitive.

   (c) a relation on $\mathbb{Q}$ that is an equivalence relation

   (d) a relation on $\mathbb{Z}$ that is transitive but not symmetric or reflexive

   (e) a relation on $\mathbb{P}_2$ that is reflexive, transitive, but not symmetric

5. Assessments

In this section, I include samples of quizzes, midterm exams, and final exams.

5.1. Linear Algebra I (MATH 3333, University of Oklahoma)

I have taught this course in *Fall 2022, Spring 2022, Spring 2021.* This course covers linear systems, Row Echelon forms, Matrices, Determinants, Inverses, Eigenvectors, Linear Transformations, Subspaces, Basis, Eigenbasis, General vector spaces like polynomial spaces, etc.

5.1.1 Midterm 1: Two parts

The next seven pages show two parts of the first midterm from Fall 2022, titled **Exam 1: Conceptual** and **Exam 1: Computational**, respectively.

5.1.2 Pop Quiz: Micro Extra Credit

I offer pop-quizzes for a small amount of extra credit, as they allow my students to test their knowledge in a low-stakes setting. The two pages after Exam 1 show Pop Quiz 12 from Spring 2022, covering a large variety of topics. This Pop Quiz was offered in the pre-finals week and students were allowed to discuss with their peers.

MATH 3333 **Exam 1: Conceptual** **September 23, 2022**

Name: ______________________________ Section : __________

INSTRUCTIONS: Please read all the questions carefully and present your solution to each problem in a clear and orderly fashion.

TOTAL POINTS = 25 **TOTAL PROBLEMS = 4**

1. (5 points) **Multiple Choice Problems. Pick one answer.**
Expectations: Please **FILL IN THE CIRCLE** next to your option. Reasoning **NOT** required.

(a) (1 point) Let A denote 4×4 matrix and assume that $\det(A) = 2$. What is $\det(-A)$?

○ $\det(-A) = 2$
○ $\det(-A) = -2$
○ $\det(-A) = -1$
○ $\det(A) = 1$

(b) (1 point) If for a 2×2 matrix A, we know that $A^2 = I$, what is A^{-1}?

○ $A^{-1} = A^2$
○ $A^{-1} = A$
○ $A^{-1} = I$
○ $A^{-1} = 0$

(c) (1 point) If A has an eigenvalue 2, then what can you guarantee for $B = A^2 + I$?

○ 5 is an eigenvalue of B
○ 2 is an eigenvalue of B
○ 7 is an eigenvalue of B
○ 4 is an eigenvalue of B

(d) (1 point) Given the matrices $A = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \end{bmatrix}$ and the fact that the product $\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$, i.e. the 1×1 identity matrix, which of the following is correct?

○ A and B are inverses of each other.
○ A and B are transposes of each other.
○ The determinant of A and B is 1.
○ A and B both have the same non-empty set of eigenvalues.

(e) (1 point) Of the four following statements, which one is TRUE?

○ Each matrix has exactly one REF equivalent to it.
○ Each square matrix has exactly one inverse.
○ It is possible to write a linear system that has exactly 4 solutions.
○ Each invertible matrix has exactly one inverse.

Solution: (a) A (b) B (c) A (d) B (e) D

2. (6 points) **Give an Example!**
Expectations: Just write your example. Reasoning **NOT** required. Please write in the space provided.

(a) (2 points) Give an example of an **inconsistent** linear system with 3 variables x, y, and z. **Please do not write an augmented matrix. Doing this will only get you half a point.**

Solution:

$$\begin{aligned} 2x + y + 3z &= 1 \\ 2x + y + 3z &= 7 \\ 5z &= 1 \end{aligned}$$

(b) (2 points) Give an example of a 3×3 **non-invertible** matrix.

Solution: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) (2 points) Give an example of a 2×2 matrix that has $\lambda = 17$ as one of the eigenvalues.

Solution: $\begin{bmatrix} 17 & 0 \\ 0 & 17 \end{bmatrix}$

3. (6 points) **Short answer question:**
Expectations: In each question, I expect you to write your answer. The reasoning should be complete but you do not have to write more than a line or so.

(a) (2 points) If for some matrix B, its inverse $B^{-1} = \begin{bmatrix} 2 & 3 \\ 6 & 7 \end{bmatrix}$, what is the inverse of B^T? **How?**

> **Solution:** $(B^T)^{-1} = (B^{-1})^T = \begin{bmatrix} 2 & 6 \\ 3 & 7 \end{bmatrix}$

(b) (2 points) Give an example of a 4×3 matrix A which has rank 1? Explain in one sentence why your matrix has rank 1. **Note: The complete reasoning has two parts.**

> **Solution:** $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ has rank 1 because it is in REF and has only one leading 1.

(c) (2 points) Consider the augmented matrix corresponding to a linear system:

$$\left[\begin{array}{cccccc|c} 1 & 0 & 7 & 0 & 3 & 1 & 1 \\ 0 & 0 & 1 & 0 & 3 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

Is this in Row Echelon Form? **Why or why not**? How many solutions does this system have? And **how do we know so**?

> **Solution:** It is NOT in REF because the rows of zeroes are supposed to be at the bottom. It has no solutions because the system is inconsistent, i.e. the rightmost column has a leading one.

4. (8 points) Are the following statements **TRUE** or **FALSE**? **Explain your answer.**
Expectations: In each part, I expect you to write your answer. The explanation should be complete but you do not have to write more than a line or so. +0.5 for T/F, +1.5 for reasoning.

(a) (2 points) The linear system in variables x and y

$$\begin{aligned} ax + by &= 0 \\ cx + dy &= 0 \end{aligned}$$

for all possible values of $a, b, c, d \in \mathbb{R}$, is consistent.

> **Solution:** TRUE. The system is consistent because it is homogeneous and have $x = 0, y = 0$ as the solution.

(b) (2 points) If two matrices A and B have the same inverse C, then $A = B$.

> **Solution:** TRUE. If $A^{-1} = C = B^{-1}$, then $A = C^{-1} = B$.

(c) (2 points) For a 2×3 matrix A, we are given $A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, so 3 is an eigenvalue of A.

> **Solution:** FALSE. The vectors $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ are not the same because of different heights. So, the equation does not represent the eigenvalue-eigenvector equation. You can also think of it this way: eigenvalues require a determinant computation and only square matrices have determinants.

(d) (2 points) For $A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \end{bmatrix}$, $BA = \begin{bmatrix} 1 & 2 \\ 1 & 2 \\ 1 & 2 \end{bmatrix}$

> **Solution:** FALSE. BA is not computable because B is of size 1×2 and A is of size 3×1.

MATH 3333 **Exam 1: Computational** **Sep 23, 2022**

Name: ______________________________ Section : ____________

INSTRUCTIONS: Present your solution to each problem in a clear and orderly fashion. You must show your work. An answer alone without supporting work **will not** receive full credit.

TOTAL POINTS = 25 **TOTAL PROBLEMS = 3**

Honor pledge: On my honor, I will maintain the highest standards of honesty, integrity, and personal responsibility. I will only use the resources that have been allowed: the cheat sheets and basic arithmetic calculators. I will not use internet search or communicate with anyone else during the exam. Write "**I have read and accepted the pledge conditions, and will abide by them.**" Failure to write this will fetch you a 0.

1. (8 points) Find all solutions to the linear system

$$\begin{aligned} 3x - 9z &= 33 \\ 7x - 4y - z &= -15 \\ 4x + 6y + 5z &= -6 \end{aligned}$$

You may use any method (**direct**, **row operations on augmented matrices**, or **inverse of coefficient matrix**) to solve this. Provide sufficient details. **Before you start, please double check that you have copied the system correctly.**

Expectations: I expect to see all the steps and all row operations listed neatly, if you use the row operation method. You can combine multiple row operations into a single step only if they are independent of each other.

Solution: We will do this problem directly. The first equation gives us $x = 3z + 11$. Substituting this into the second and third equations we get

$$\begin{aligned} 7(3z + 11) - 4y - z &= -15 \\ 4(3z + 11) + 6y + 5z &= -6 \end{aligned}$$

This simplifies to

$$\begin{aligned} -4y + 20z &= -92 \\ 6y + 17z &= -50 \end{aligned}$$

The first equation becomes $y = 5z+23$, which we can substitute into the second one. This gives us $6(5z + 23) + 17z = -50$ which simplifies to $z = -4$. Using that we get $y = 5(-4) + 23 = 3$ and $x = 3(-4) + 11 = -1$. The linear system has a unique solution $(-1, 3, -4)$.

2. (8 points) Compute the determinant of $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 10 \\ 3 & 0 & 3 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$ using your favorite method. **Provide sufficient details. Before you start, please double check that you have copied the matrix correctly.**

Expectations: I expect to see all the steps and all row operations listed neatly, if you use the row operation method. You can combine multiple row operations into a single step only if they are independent of each other.

Solution: We know that

$$\det \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 10 \\ 3 & 0 & 3 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix} = \det \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 3 & 0 & 3 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

because the two matrices are related by the row operation: Add -2(Row 4) to Row 2. The determinant on the right can be computed by just taking a product of the diagonal entries since the matrix is lower triangular. Therefore, $\det(A) = 1(2)(3)(5) = 30$.

3. (9 points) Given a matrix $B = \begin{bmatrix} -2 & 0 & 4 \\ 0 & 1 & 0 \\ 2 & 0 & 5 \end{bmatrix}$, and the fact that $\lambda = 1$ is an eigenvalue of B, compute the set of all eigenvectors of B. **Before you start, please double check that you have copied the matrix correctly.**

Expectations: Set up things neatly, and then solve the system for eigenvectors, conclude your answer with all the details.

Solution: To find the eigenvectors, we first need to compute the matrix:

$$\lambda I - B = \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 0 & 4 \\ 0 & 1 & 0 \\ 2 & 0 & 5 \end{bmatrix} = \begin{bmatrix} \lambda + 2 & 0 & -4 \\ 0 & \lambda - 1 & 0 \\ -2 & 0 & \lambda - 5 \end{bmatrix}$$

which becomes for $\lambda = 1$:

$$\begin{bmatrix} 3 & 0 & -4 \\ 0 & 0 & 0 \\ -2 & 0 & -4 \end{bmatrix}$$

We now set up the system $(\lambda I - B)\vec{v} = \vec{0}$ in equation form:

$$\begin{aligned} 3x + 0y - 4z &= 0 \\ 0x + 0y + 0z &= 0 \\ -2x + 0y - 4z &= 0 \end{aligned}$$

Since all coefficients of y are 0, it can be represented by a real parameter t. Solving the other equations gives us $x = 0$ and $z = 0$. Therefore, the set of eigenvectors for $\lambda = 1$ is:

$$\left\{ \begin{bmatrix} 0 \\ t \\ 0 \end{bmatrix} \middle| \ t \text{ is a non zero real number} \right\}$$

MATH 3333 **Pop Quiz 12** **Spring 2022**

Name: ________________________________ Section : __________

INSTRUCTIONS: Present your solution to each problem in a clear and orderly fashion.

1. (4 points) What is the dimension of the following spaces?

   (a) $\dim(\mathbb{R}^n) =$ __________

   (b) $\dim(\mathbb{P}_1) =$ __________

   (c) $\dim(\mathbb{P}_n) =$ __________

   (d) $\dim(\mathbb{P}) =$ __________

   (e) $\dim(\mathbb{S}) =$ __________

   (f) $\dim(\mathcal{C}^\infty) =$ __________

   (g) $\dim(\mathbb{R}^{3\times 2}) =$ __________

   (h) $\dim(\mathbb{R}^{4\times 4}) =$ __________

2. (3 points) Complete the basis by adding elements!

   (a) Add vectors to $\left\{ \begin{bmatrix} 7 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} \right\}$ to make it a basis for $\mathbb{R}^3$.

   (b) Add vectors to $\{3x^3, 4x^4, 5x^5, 6x^6\}$ to make it a basis for $\mathbb{P}_6$.

   (c) Add vectors to $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix} \right\}$ to make it a basis for $\mathbb{R}^{2\times 2}$.

3. (2 points) Linear Independent or not?

   (a) If $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is independent, then $\{\mathbf{x}, \mathbf{y}\}$ is independent.

   (b) If one of $\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{w}$ is zero, then $\{\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{w}\}$ is dependent.

4. (2 points) My doctor wants to give me 84 mg of vitamin C and 130 mg of vitamin D per day. She has two supplements: the first contains 10% vitamin C and 25% vitamin D; the second contains 20% vitamin C and 25% vitamin D. To find out how much of each supplement should she give me each day, we need a system of linear equations. Convert this word problem into that linear system of equations. **CONVERT BUT DON'T SOLVE! Specify your variables.**

5. (1 point) Describe the physical effect of the linear transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$ given by:

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$$

5.2. Calculus & Analytic Geometry (MATH 2443, University of Oklahoma)

I have taught this course in *Fall 2021, Fall 2020.* This course covers multivariable differentiation, tangent planes, chain rule, gradient, maximum and minimum, double integration, polar integration, cylindrical coordinates, spherical coordinates, divergence, curl, Green's theorem, Stoke's theorem, Divergence theorem, etc.

5.2.1 Quiz on Double Integrals and Polar Integrals

The next three pages shows Quiz 05 from Fall 2021. This quiz is based on the topic of double integrals and polar integrals. The assignment is accompanied by the solutions.

5.2.2 Final Exam

The two pages after Quiz 05 show the Final Exam from Fall 2021.

MATH 2443 **Quiz 5** **Due at 10/06/2021, 11:59 PM**

Name: ______________________________ Section : __________

INSTRUCTIONS: Present your solution to each problem in a clear and orderly fashion. You must show your work. An answer alone without supporting work **will not** receive full credit. Upload the solutions to Gradescope before the deadline. **You have 30 minutes to solve AND upload.**

TOTAL POINTS = 10 **TOTAL PROBLEMS = 2**

1. (5 points) Evaluate $\iint\limits_D y\, dA$ where D is the triangular region with vertices $(0,0)$, $(1,1)$, and $(1,0)$.

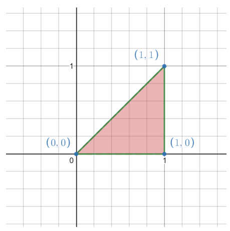

Figure 1: From Question 1. The region D.

Solution: We first write the domain:

$$D = \{(x,y) \mid 0 \le x \le 1, 0 \le y \le x\}$$

Now, we double integral in the iterated integral form:

$$\iint\limits_D y\, dA = \int_0^1 \int_0^x y\, dydx$$

Then we integrate with respect to y first and then with respect to x:

$$\int_0^1 \int_0^x y\, dydx = \int_0^1 \left[\frac{y^2}{2}\right]_0^x dx = \int_0^1 \left(\frac{x^2}{2}\right) dx$$
$$= \left[\frac{x^3}{(2)(3)}\right]_0^1 = \frac{1}{6}$$

Therefore,

$$\iint_D y \, dA = \frac{1}{6}$$

2. (5 points) **Change to polar coordinates and evaluate**: $\iint_D \cos(\sqrt{x^2+y^2}) \, dA$ where D is the region in the upper half plane enclosed between the circles $x^2+y^2 = 4$ and $x^2+y^2 = 9$.

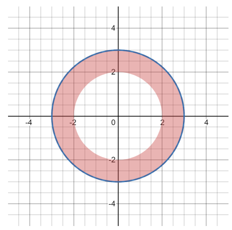

Figure 2: From Question 2. The region D.

Solution: To convert into polar coordinates, we need to convert the integrand and write D in polar coordinates. First, the integrand can be converted as:

$$\cos(\sqrt{x^2+y^2}) = \cos(\sqrt{x^2+y^2}) = \cos(r)$$

Since the domain is the annulus with radii 2 and 3 in the upper half plane,

$$D = \{(r,\theta) \mid 2 \le r \le 3 \ , \ 0 \le \theta \le \pi\}$$

Now, we can convert the integral and integrate:

$$\iint_D \cos(\sqrt{x^2+y^2}) \, dA = \int_0^\pi \int_2^3 \cos(r) r \, dr d\theta$$

To find the anti-derivative of $\cos(r)r$, we use Integration by parts using $u = r$ and $dv = \cos(r)$:

$$\int udv = uv - \int vdu = r\sin(r) - \int \sin(r)(1)dr = r\sin(r) + \cos(r)$$

Page 2

So the integral becomes

$$\int_0^\pi \int_2^3 \cos(r)r \; dr d\theta$$

$$= \int_0^\pi [r\sin(r) + \cos(r)]_2^3 \; d\theta = \int_0^\pi (3\sin(3) + \cos(3) - 2\sin(2) - \cos(2)) \; d\theta$$

$$= [(3\sin(3) + \cos(3) - 2\sin(2) - \cos(2))\,\theta]_0^\pi = (3\sin(3) + \cos(3) - 2\sin(2) - \cos(2))\,\pi$$

Therefore,

$$\iint_D \cos(\sqrt{x^2+y^2}) \; dA = (3\sin(3) + \cos(3) - 2\sin(2) - \cos(2))\,\pi$$

Page 3

MATH 2443 **Final Exam** **12/13/2021**

Name: ______________________________ Section : __________

INSTRUCTIONS: Present your solution to each problem in a clear and orderly fashion. You must show your work. An answer alone without supporting work **will not** receive full credit. Upload the solutions to Gradescope before the deadline. **You have 110 minutes to solve AND 10 minutes to upload.**

TOTAL POINTS = 100 **TOTAL PROBLEMS = 11**

Short-ish Problems

1. (10 points) **One word/One Sentence problems:**

   (a) (2 points) For a function f, we compute the directional derivatives of f at a given point in the direction of the vectors $\vec{u}$ and $c\vec{u}$ where $c > 0$ is a scalar. How are $D_{\vec{u}}f$ and $D_{c\vec{u}}f$ related to each other?

   (b) (2 points) I am given a surface $\phi = \frac{\pi}{2}$ in spherical coordinates (ρ, θ, ϕ). Describe the shape! **Hint: It is not a cone.**

   (c) (2 points) Given a conservative vector field $\mathbf{F}$ and a closed loop C traversed only once, what can you say about $\int_C \mathbf{F} \cdot d\mathbf{r}$?

   (d) (2 points) For a function f, what kind of a critical point do we get if the double derivatives at that point satisfy $f_{xx} = 1$ and $f_{yy} = -2$?

   (e) (2 points) I am interested in the evaluation $f(1.01, 1.99)$ of an unknown function $f(x, y)$. However, you are given its linearization function $L(x, y)$ at $(1, 2)$ and the evaluation $f(1, 2)$. How can you help me find an approximation to $f(1.01, 1.99)$?

2. (7 points) Parameterize the following (**do not forget the bounds!**):

   (a) (3 points) the line segment from $(1, 2, 3)$ to $(3, 4, 5)$.

   (b) (4 points) the surface $x^2 + y^2 = 9$, for $1 \leq z \leq 3$.

3. (9 points) **Short answer questions:**

   (a) (4 points) **Express** the domain of $f(x, y, z) = \sqrt{4 - x^2} + \sqrt{9 - y^2} + \sqrt{1 - z^2}$ as a set in $\mathbb{R}^3$.

   (b) (5 points) Find $\frac{\partial z}{\partial t}$ if $z = \sin\left(\frac{x}{y}\right)$ where $x = s + t$, and $y = s - 2t$. **Your final answer must be in terms of** s **and** t.

4. (6 points) We are given a point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1\right)$ in Cartesian coordinates. Change it to both **spherical coordinates** and **cylindrical coordinates**. **Show work!**

5. (6 points) Is there a vector field $\mathbf{F}$ such that $curl(\mathbf{F})(x, y, z) = \mathbf{G}(x, y, z) = \langle x^3, y^3, z^3 \rangle$? **Why or why not? Show work!**

Standard Problems

6. (8 points) Show that the limit $\lim\limits_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$ does not exist. **Caution: Do not forget to write limits as you compute things.**

7. (10 points) Find the area of the region D bounded by one loop of the curve $r = \sin(\theta)$, given in polar coordinates.

8. (10 points) Change to cylindrical coordinates **but do not integrate**: $\iiint\limits_E \ln(x^2+y^2)\, dV$ where E is the solid that lies **inside** the cylinder $x^2 + y^2 = 1$, **above** the plane $z = 0$, and **below** the cone $z^2 = 4x^2 + 4y^2$.

9. (10 points) Use the Divergence theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ (that is, calculate the flux of $\mathbf{F}$ across S), where $\mathbf{F}(x, y, z) = \langle xye^z, xy^2z^3, -ye^z \rangle$ and S is the box bounded by the coordinate planes and the planes $x = 3$, $y = 2$, $z = 1$.

10. (10 points) Use Stokes' theorem to evaluate $\iint_S curl(\mathbf{F}) \cdot d\mathbf{S}$ where

$$\mathbf{F}(x, y, z) = \langle \tan^{-1}(x^2yz^2), x^2y, x^2z^2 \rangle$$

and S is the part of the cone $x = \sqrt{y^2 + z^2}$, $2 \geq x \geq 0$, oriented in the direction of positive x-axis.

11. (14 points) Find the absolute maximum and minimum values of $f(x, y) = x^2+y^2+x^2y+2$ on the domain $D = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$.

    (a) (4 points) Find all the critical points of f. How many (and which ones) of these are inside D (that is, not on the boundary).

    (b) (8 points) Find all the absolute maximum and minimum values of f on the boundary of D.

    (c) (2 points) Using (a) and (b), write the absolute maximum and minimum values of f on D.

5.3. Transitions to Proofs (MTH 299, Michigan State University)

I have taught this course in *Fall 2019, Fall 2018, Summer 2018, Spring 2017*. This course serves as an introduction and transition to proof-writing in Mathematics. The students enrolled in this course were mainly mathematics majors and minors, and they take it before they take advanced mathematics courses. The course content includes propositional logic, elementary set theory, proof techniques such as contradiction and induction, and some applications of the aforementioned in real analysis and number theory.

5.3.1 Final Exam

The next nine pages show the Final Exam from Fall 2018. The final exam was written by all the instructors, including the course coordinator, in a collaborative effort.

Name: ______________________________

Section: ______________ **Instructor**: ______________________________

READ THE FOLLOWING INSTRUCTIONS.

- **Do not open your exam until told to do so.**
- No calculators, cell phones or any other electronic devices can be used on this exam.
- Clear your desk of everything except pens, pencils and erasers.
- If you need scratch paper, use the back of the previous page.
- Without fully opening the exam, check that you have pages 1 through 9.
- Fill in your name, etc. on this first page.
- **Show all your work.** Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand and a proctor will come to you.
- There is no talking allowed during the exam.
- You will be given exactly 80 minutes for this exam.

I have read and understand the above instructions: ______________________________

SIGNATURE

Short "Answer Only" Questions.

1. (6 points) Assume T is a partition of $\mathbb{R}$, where $T = \{A, B, C, D, E\}$, and A, B, C are given as:

$$A = \{x \in \mathbb{R} \mid x \leq -47\}$$
$$B = (2, 4)$$
$$C = \{4\}$$

Give a choice of sets D and E so that T is a partition of $\mathbb{R}$.

$$D = \underline{(4, \infty)}$$
$$E = \underline{(-47, 2]}$$

2. (8 points) For each $n \in \mathbb{N}$, let $A_n = \left[-1 + \frac{1}{n}, 4 + \frac{1}{n}\right)$. Compute the sets

$$\bigcup_{n \in \mathbb{N}} A_n = \underline{(-1, 5)}$$
$$\bigcap_{n \in \mathbb{N}} A_n = \underline{[0, 4]}$$

3. (9 points) Give an example of the following:
Note: You DO NOT have to provide any proof for any of the following.

(a) A relation on the set $\{a, b, c\}$ that is reflexive and symmetric but not transitive.

Solution: The relation could be one that only returns a true value for: $a \sim a$, $c \sim c$, $b \sim b$, $a \sim b$, $b \sim a$, $b \sim c$, and $c \sim b$.

(b) A partition of $\mathbb{R}$ where each set in the partition is unbounded.

Solution: one example could be $A_1 = \mathbb{Q}$ and $A_2 = \mathbb{R} \setminus \mathbb{Q}$.

(c) A bounded sequence of real numbers that is not convergent.

Solution: A fun example could be $a_n = \sin(n) - 5$ (but to prove it is not convergent is actually rather hard!). You could also have tried $a_n = (-1)^n - 500$.

Full Justification Questions. Provide complete justifications for your responses.

4. (10 points) Prove that if $n \in \mathbb{N}$ then $\gcd(n, 13) = 13$ or 1.

Solution: Assume that $g = \gcd(n, 13)$. This means that we have

$$g|n \quad \text{and} \quad g|13.$$

Since 13 is prime and $g|13$, we know immediately that the only candidates for g are

$$g = -13, \; g = -1, \; g = 1, \; g = 13.$$

Finally, since g is a GCD (of anything!), we know that we always have $g \geq 1$. Thus, the only options are $g = 1$ or $g = 13$.

5. (20 points) The relation $\sim$ on the set $\mathbb{Z}$ is defined by $a \sim b \iff (a+3) \equiv (b-2) \mod 5$.

   (a) Prove that $\sim$ is an equivalence relation.

   (b) Describe all the distinct equivalence classes of $\sim$. NO PROOF REQUIRED.

Solution: In order to show that $\sim$ is an equivalence relation, we will show that it satisfies the following 3 properties.
Reflexivity ($a \sim a$): $a+3-(a-2) = 5 = 5(1)$ and so $a+3 \equiv (a-2) \mod 4$
Symmetry (If $a \sim b$ then $b \sim a$):
If we know that $a \sim b$ then

$$\begin{aligned}
& a+3 \equiv (b-2) \mod 5 \\
&\implies (a+3)-(b-2) = 5k \text{ for some } k \in \mathbb{Z} \\
&\implies a-b = 5(k-1) \\
&\implies b-a = 5(-k+1) \\
&\implies (b+2)-(a-3) = 5(-k+2) \\
&\implies (b+2) \equiv (a-3) \mod 5 && (\text{since } -k+2 \in \mathbb{Z})
\end{aligned}$$

So we have $b \sim a$ which mean $\sim$ is symmetric.
Transitivity: (If $a \sim b$ and $b \sim c$ then $a \sim c$)
Suppose $a \sim b$ and $b \sim c$. Then $a+2 \equiv (b-3) \mod 5$ and $b+2 \equiv (c-3) \mod 5$, i.e. $(a+2)-(b-3) = 5k$ and $(b+2)-(c-3) = 5m$ for some $k, m \in \mathbb{Z}$.
Adding the above two equations, we get $(a+2)-(c-3)+5 = 5(k+m) \implies (a+2)-(c-3) = 5(k+m-1)$
Since $(a+2)-(c-3) = 5l$ where $l \in \mathbb{Z}$ since $l = k+m-1$, this means $a+2 \equiv (c-3) \mod 5$ and so $a \sim c$, thus proving $\sim$ is transitive.

Now, in order to find the **equivalence classes**, we first note that
$a+3 \equiv (b-2) \mod 5 \implies (a+3)-(b-2) = 5k$ for some $k \in \mathbb{Z}$
$\Leftrightarrow a-b = 5(k-1) \Leftrightarrow a \equiv b \mod 5$

Thus, we see that there are 5 equivalence classes –
$[0] = \{5k | k \in \mathbb{Z}\}$
$[1] = \{5k+1 | k \in \mathbb{Z}\}$
$[2] = \{5k+2 | k \in \mathbb{Z}\}$
$[3] = \{5k+3 | k \in \mathbb{Z}\}$
$[4] = \{5k+4 | k \in \mathbb{Z}\}$

6. (19 points) Let $a_n = \dfrac{2n^3}{3n^3+5n}$.

   (a) State the limit that the sequence $\{a_n\}_{n\in\mathbb{N}}$ converges to.

   Solution: The sequence $\{a_n\}_{n\in\mathbb{N}}$ converges to $\dfrac{2}{3}$.

   (b) Using the definition of convergence, prove that the the sequence $\{a_n\}_{n\in\mathbb{N}}$ converges to the limit from part (a).

   Solution: Let $\epsilon > 0$ be given. Since $\epsilon \in \mathbb{R}$, and $\epsilon \neq 0$, $\sqrt{\dfrac{10}{9\epsilon}} \in \mathbb{R}$. By the Archimedean property, $\exists N \in \mathbb{N}$ such that $N > \sqrt{\dfrac{10}{9\epsilon}}$. Now $\forall n \geq N$ we have $n \geq N > \sqrt{\dfrac{10}{9\epsilon}} \implies \dfrac{10}{9n^2} < \epsilon$.
   So $\forall n \geq N$, we have

$$\begin{aligned} |a_n - L| &= \left|\frac{2n^3}{3n^3+5n} - \frac{2}{3}\right| \\ &= \left|\frac{6n^3 - 6n^3 - 10n}{9n^3+15n}\right| \\ &= \frac{10n}{9n^3+15n} < \frac{10n}{9n^3} = \frac{10}{9n^2} < \epsilon \end{aligned}$$

   Thus, we have proved that the sequence $\{a_n\}_{n\in\mathbb{N}}$ converges to $\dfrac{2}{3}$.

7. (18 points) Prove that $\forall n \in \mathbb{Z}$, $(n^2 + n) \not\equiv 1 \mod 3$.

Solution: We assume that $n \in \mathbb{Z}$ is generic. The division lemma, with a divisor of 3, gives a unique representation of n as

$$n = 3k + r, \quad \text{for a unique choice of } k \in \mathbb{Z} \text{ and } r \in \{0, 1, 2\}.$$

Next, we will make a simplification on checking $n^2 + n$:

$$n^2 + n = (3^2k^2 + 6kr + r^2) + (3k + r) = 3m + r^2 + r, \quad \text{for } m = 3k^2 + 6kr + k.$$

We need to demonstrate that 3 *does not* divide $n^2 + n - 1$.

Our calculation demonstrates that

$$3|(n^2 + n - 1) \iff 3|(r^2 + r - 1).$$

Thus, we can conclude by checking the four unique cases for r:

$$\begin{aligned} r = 0 &\implies r^2 + r - 1 = -1 \\ r = 1 &\implies r^2 + r - 1 = 1 \\ r = 2 &\implies r^2 + r - 1 = 5 \end{aligned}$$

In all cases, we see that 3 does not divide $r^2 + r - 1$. Thus, we have confirmed that

$$n^2 + n \not\equiv 1 \mod 3.$$

8. (12 points) Prove that the set $E = \{3^k | k \in \mathbb{N}\}$ is unbounded.

Solution:

We will demonstrate that the negation of the definition of bounded is true. Thus, assume that $M \in \mathbb{R}$ is given and generic. We also assume that $M > 0$.

SCRATCHWORK: We want to find $x \in E$ with $|x| > M$, i.e. we need some $n \in \mathbb{N}$ so that $3^n > M$. Solving for n gives that $n > \log_3(M)$.

PROOF: Recall that $M \in \mathbb{R}$, $M > 0$. This implies that $\log_3(M)$. By the Archimedean property there exists some $n \in \mathbb{N}$ so that $\log_3(M) < n$. Let $x = 3^n$. Then $x \in E$ and $|x| = 3^n > 3^{\log_3(M)} = M$.

Thus, we have demonstrated the existence of $x \in E$ with $|x| > M$. Since M was generic, we conclude that E is not bounded.

Congratulations you are now done with the exam!
Go back and check your solutions for accuracy and clarity.

When you are completely happy with your work please bring your exam to the front to be handed in.
Please have your MSU student ID ready so that is can be checked.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	14	
3	9	
4	10	
5	20	
6	19	
7	18	
8	12	
Total:	102	

v1

6. Evidence of Teaching Effectiveness: Student Comments

6.1. Linear Algebra I (MATH 3333, University of Oklahoma)

I have taught this course in *Fall 2022, Spring 2022, Spring 2021.* This course covers linear systems, Row Echelon forms, Matrices, Determinants, Inverses, Eigenvectors, Linear Transformations, Subspaces, Basis, Eigenbasis, General vector spaces like polynomial spaces, etc. Here are some of the comments from student evaluations of the class over the 2 completed semesters of the class.

- I believe Dr. Gakhar is great professor, He has always should interest in students lives and always shows he cares about his students. Linear algebra is not a simple subject but me makes it so engaging and fun to learn almost like solving a puzzle. By far one of my favorite professors at OU.... Give him a raise on his pay check because he is a real gem.
- Dr. Gakhar is great at making students feel welcome in his class! I genuinely appreciate how although he does pick on people to answer questions, when I told him in a survey that I get bad anxiety from randomly being called on, he no longer did it. I felt as though I was able to focus a bit better from there on out rather than be a bit nervous that he may call on me.
- All the notes and the online videos for this class were very clear and concise. The examples included were helpful with understanding the material and the difficulty of the homework and quizzes were very helpful with preparing for the exams. Overall, I have no complaints.
- I felt Dr. Hitesh was very helpful and was always open to help. He was very understanding and would adjust dates based on where the class was at in terms of scheduling and understanding.
- He is the best math professor here at OU. Homework feels very purposeful and in-class explanations are very clear.
- I liked how we were treated as though we were humans and not homework-doing machines. We were always given extensions when needed and even though we technically had a schedule in the beginning, it changed quickly on, but only for the better of the students. If we did not get a chance to turn something in on time, we would still be able to submit it just fine. He always listened to our suggestions and seemed to actually implement them as well, rather than claiming that he would. 10/10 professor– there is nobody like him in the math department, or honestly the entire university, for sure.
- I think this class was excellent overall, and I really liked the class structure of learning the material outside of class and doing examples in class. There's always the drawback that we had to spend more time outside of class than normal, but I think the opportunity to see more problems worked out in class is a good tradeoff.
- Linear Algebra is the best upper division math class I've taken and it is because of the instructor. When I talk to friends in other sections, I find that I am having a much better experience and I understand the material more than my peers.
- Honestly I don't know. Dr. Gakhar is the best professor I've had in the math department. The Math department desperately needs to find and hire more people like him. I honestly hate the math department and I really wish I could have Dr. Gakhar for all of my classes moving forward, I haven't had a professor in the math department that cares about their students as much as he does.
- Overall, this was an awesome class. It was easily my best experience with the math department at OU. Even just seeing Dr. Gakhar say "Happy Learning" at the end of the episodes made me very happy with the class, even though it was such a small thing. I tried to take linear algebra last semester, and I dropped in the second week because I was not understanding any of the proofs and concepts. Trying the course with Dr. Gakhar this semester completely turned that around. I am so glad I took this class. If I see Dr. Gakhar teaching any future courses that I need, I will immediately enroll. I loved this class.

- The notes were super clear, as were the associated episodes. I thought Dr. Gakhar did a great job of explaining all of the concepts and answering questions in class. He very clearly cares about the students and their understanding of the material. The homeworks were also a very reasonable length. I would always get them done within one day without too much trouble. Even though they were fairly short, with most being only a few problems, I never felt underprepared for anything.
- I learned a lot of material in this course. Your willingness to listen to student feedback with polls and in class as well as pushing deadlines is very appreciated. There was a point where I was drowning in coursework and an extended deadline helped me take lots of stress off of my back. Also, the worksheets helped out a lot with understanding the material and I'm glad that you had worksheets every week.
- I wish I had a professor like Dr. Hitesh for all my other prior math courses. I had him for calculus 4 and now linear algebra. I notice a HUGE difference in my performance compared to prior math courses where Dr. Hitesh wasn't my professor. I have both a physical and learning disability and I truly believe that Dr. Hitesh has the best course structure for those like me to succeed. I understand the material so much better than I once did.
- The professor being patient in listening to the students' suggestions and frequently asking questions rather than only giving questions is impactful in students' perspectives and efforts in learning. Additionally, he posts many announcements to keep students updated for their success in the course. The pacing of the material was also good for understanding once I (and I believe everyone else) got used to the class. I disliked math because I'm clumsy and make many small mistakes that does not result in the correct solution, but the course began to be fun for me and although I'm not great at it, it was an enjoyable course.
- 1) High-quality lecture slides, class notes, and worksheets significantly contributed to my understanding. 2) The use of notes that are allowed on exams allowed me to prove my understanding of the material more realistically. 3) Dr. Hitesh really cares about student feedback and makes decisions based on our feedback that almost always best helped us.

6.2. Calculus & Analytic Geometry (MATH 2443, University of Oklahoma)

I have taught this course in *Fall 2021, Fall 2020.* This course covers multivariable differentiation, tangent planes, chain rule, gradient, maximum and minimum, double integration, polar integration, cylindrical coordinates, spherical coordinates, divergence, curl, Green's theorem, Stoke's theorem, Divergence theorem, etc. Here are some of the comments from student evaluations of the class over the 2 semesters of the class.

- Dr. Gakhar was the strong point of this course. Hands down, the absolute BEST professor in the math department. The math department is in desperate need of more professors like Dr.Gakhar. I truly believed I was stupid and entirely incapable of understanding calculus for the longest time until this calculus 4 course. I took calculus 1 twice, calculus 2 three times, and calculus 3 once, but I struggled greatly in all of those times. This past semester in calculus 4 with Dr. Gakhar taught me that I could understand and perform calculus. He treats his students with kindness, compassion, and respect and does an excellent job explaining topics, answering questions, and grading fairly. I can't express how happy I am to feel like I finally understand it all better, and it really is thanks to Dr. Gakhar.
- Dr. Gakhar didn't let us get away with pretending to know the material, even in a Zoom lecture he managed to challenge our knowledge daily. He was always concerned about our well-being and our understanding as students, I felt like my opinion always mattered and he made sure that it did. Even though I did my homework on-time and hardly needed extensions, he was always willing to extend homework. He is the kind of professor that if you are in a bad place you can go to him and discuss how you can fix your grade and he will try his best to help you.
- Gakhar was extremely good at engaging students, despite it being via zoom. I really liked that he devoted most of class time to problem solving, and we watched short lecture videos outside of class time. Despite having usually 40 or so people in each lecture, Gakhar was still really good at engaging each student and encouraging in class participation.

- Professor Gakhar's understanding of the course material and the underlying math behind its sometimes seemingly abstract equations is very impressive and when probed about the inner workings of said equations he explains it fairly clearly. That type of granular detail, underlying logic, and graphical representation of math is how I best learn the subject and for that cause he was very very good. Moreover, his investment in making sure the class was all together on its understanding was commendable! Although it took time out of the class, his habit of asking the class frequently if everyone understood was comforting. He clearly did not want to see his students fall behind because he knows that math is cumulative and that being weak in basic concepts can doom a student's understanding of complex concepts. Also, his outside of class videos and PDFs in conjunction with the thorough explanation of subjects during lecture was a great format for cementing concepts in my head.
- Overall, I have never enjoyed a calculus course so much. Dr. Gakhar is a very clear and effective teacher while making the class interesting and enjoyable at the same time.
- This professor was awesome. He encouraged independent learning and never made us feel inadequate.
- One of the strong points was the flipped classroom. Dr. Gakhar uploads videos to canvas that explain what we will be learning in class and provides many other online resources. I found this to be really helpful. He is also very kind and a great teacher. He explains things clearly and always tries to make sure we understand before moving on.
- I've enjoyed this class a lot. Dr. Gakhar has done a great job and really cares about his students.
- Dr. Gakhar was great about giving relevant assignments and making sure everyone had a chance to learn from them.
- It was a really enjoyable course with a enjoyable professor. I felt like the prof truly cared about us and made sure we all did well. This was by far the best interaction I've had with the math department and a highlight of my math career.
- The amount of care and work put into the class made me fully appreciate the professor and the course workload. The way the class was taught made the material very understandable and enjoyable.
- I loved it and really appreciated my instructor's flexibility on homework/ quiz deadlines. I also like how he assigned deadlines according to what the class wanted.
- I thoroughly enjoyed this class and feel confident about the material that I have learned. I can say with confidence that Professor Gakhar is probably one of the best math professors I have had at OU. The course style, although flipped was actually extremely beneficial to my learning. Viewing the lectures outside of class time and working on problems in class allows for more time to ask questions and really understand the content that is being presented throughout the semester. As someone who enjoys math, but has always struggled with it, Professor Gakhar has helped me gain confidence in my mathematical abilities and has contributed greatly to my learning.
- The teaching style is really beneficial to students and allows lots of room to ask questions and understand the content. Professor Gakhar also cares deeply about his students and makes sure that there is enough time to complete each assignment on time, offering extensions if needed. He also values the opinions of the students and takes time to make sure that the class feels confident about the material.
- The entire class was amazing. The reversed classroom style worked great for me, the assignments were very difficult but the resources were clear and helpful. The leniency with due dates and understanding that we don't have a lot of time was very appreciated.
- There are many strong points. They include working with the students and checking in with them on how they are doing in life and with the course, providing videos before class within the flipped style of learning, the use of written homework rather than solely relying on WebWork was extremely helpful, and most importantly the equations sheets provided for each test was the absolutely most helpful aspect of the entire course for me. That aspect really represents real life in that we will have access

to any equation we will need. So it helps prepare us for work after college and helps de-emphasize the stress of memorizing all types of various equations that we will just forget after the temporary test due to how short-term memory works. The aspect of working with the students throughout the semester in all aspects of the course was great. He is also very personable and approachable which is really nice.

- Dr. Gakhar is very dedicated to what he's doing and he's a very kind and understanding professor. The videos for class were super helpful and I learned a lot in this course, specifically due to the nature of the lessons and how they were all given an ample amount of time in lecture.

- Professor Gakhar is the highlight of the course. He is a fantastic teacher. His communication (both in class and outside) is excellent. My canvas inbox is completely full of messages from him with a VERY light scattering of messages/announcements from my other teachers - and that is a testament to his commitment to good communication. While he treats us like adults (he never talks down to us), he is also very helpful, giving reminders of due-dates, expectations, and consistent updates regarding our learning roadmap. I also really appreciate his flipped-classroom approach. I like that I can watch and re-watch his videos to attempt to master a topic, and he expounds on the videos in class, as opposed to just repeating what we've already seen. I would, without hesitation take another one of his courses. I also think that canvas polls are a smart way to gauge participation in online classes (other teachers require active participation in the chat window). While I participate quite a bit, it's not a very natural participatory environment, so the polls are a good idea. And finally, when he says, "is this clear to everyone, any questions?", etc., he actually pays attention to the chat window and expects people to respond, making them accountable for their learning/non-learning. Sure, most people say "clear", even if they don't really mean it (I'm occasionally guilty of this), but it puts the onus on the student.

- Feedback loops were great and let the professor really understand where we were in our understanding. the way the course was structured was amazing, more math teachers need to follow the same formula The way he pushed for our participation was truly inspiring

- - Great in-class experience - Professor Gakhar was genuinely wanting to provide his students with the best learning experience possible - feedback was requested and taken into consideration - understanding of external conflicts - flipped class experience

6.3. Transitions to Proofs (MTH 299, Michigan State University)

I have taught this course in *Fall 2019, Fall 2018, Summer 2018, Spring 2017.* This course serves as an introduction and transition to proof-writing in Mathematics. The students enrolled in this course were mainly mathematics majors and minors, and they take it before they take advanced mathematics courses. The course content includes propositional logic, elementary set theory, proof techniques such as contradiction and induction, and some applications of the aforementioned in real analysis and number theory. Here are some of the comments from student evaluations of the class over the 4 semesters of the class.

- I enjoyed taking the course and expanding my abilities and perspective of mathematics! Challenging to keep up initially, but the continuous deadlines eventually made the final 2/3 of the course flow much better. Notation was tough to remember at first, but once I had it down, writing problems and solutions became much more efficient. The constant usage of the symbols helped engrain them in my brain! Overall, a wonderful experience for a high school student!

- Yes! At first it was hard to keep up with Hitesh because he went right into it but I am very grateful he did that because it made learning in the future much easier and made me understand the notation very well. I enjoyed the practice exam reviews given and questions Hitesh would give us.

- He's a smart instructor. But most of the times I feel like his way of explaining is not organized as he expects the students to think as fast as him. But he is a very nice person.

- I would definitely recommend Hitesh for not only is he knowledgeable and a competent teacher, but he puts forth a lot of effort in his classroom. He learned everyone's name very quickly which is sadly very rare. He was very engaging and entertaining as he taught. He took our feedback and requests and wrote us a practice exam when we asked for one. He brought us candy! I think sometimes he struggled to understand questions students were asking, but he compensated for that by being patient and having them write on the board to explain their ideas. I do think that the general course set up made it difficult to know where your knowledge gaps were until too late almost. I think Hitesh could have aided that by having the class instruct him on how to do a problem instead of watching him solve it (ie. what do we do now?, and next what do we do?). He did do this sometimes. I also think that group work in class and homework due after the weekend wasn't a great schedule. This contributed to the fact that sometimes I wasn't sure if I knew what I was doing on my own. Having two smaller homeworks due at different times would encourage students to work more efficiently. Also we were told that he or (TA) did not want to upload everything in terms of grades to D2L because it takes too long, which frustrated some people and if that's the case then there should be another place to communicate grades.

- I would recommend Hitesh to another student. He presented the material in an easy to understand manner and always made sure everyone in the class understood the material and if they didn't he would answer their questions in a very clear and understanding way. The course material gets a little boring but Hitesh made it interesting. Additionally, he was pretty funny and would make jokes to light up the mood.
- Yes Hitesh has been my favorite teacher at MSU and if I could have him teach all my math classes I would.
- I would definitely reccommend Hitesh. He is a great instructor that keeps you engaged.
- I would recommend to other students. Hitesh was able to clearly communicate difficult concepts in several different ways when one way was unclear. When he saw a homework question that was consistently interpreted in the wrong way, he made sure to bring it up in class and explain it.

7. Evidence of Teaching Effectiveness: Numerical Summaries

In this section, I provide details of my average numerical ratings. The tables are color-coded for to offer a big picture perspective, with **Green** representing the best performance scores, and **Maroon** representing the worst performance scores. Exact color-keys are provided on each page since OU and MSU used different scales.

7.1. University of Oklahoma: New System

In Spring 2022, OU's Teaching Evaluation system *eValuate* was replaced by the *Student Experience Survey*, where they moved away from numerical ratings.
In Spring 2022 semester, I taught Linear Algebra and at a 42% response rate (26 out of 62), about:

- 90.3% students felt that *the expectations and grading scheme for assignments and the overall course were made* **very clear**.
- 96.2% students felt that I **frequently** *appeared to be organized and prepared for student interactions.*
- 92.3% students felt that *the course website/Canvas was* **very helpful**.
- 84.6% students felt that *the reading assignments and/or online instructional materials were beneficial to their learning* **frequently**.
- 80.7% students felt that I **frequently** *explained or demonstrated the larger purpose or value of the material covered in this course.*
- 84.6% students felt that I **frequently** *used a variety of examples or approaches to illustrate concepts.*
- 100% students felt **frequently** that *the assignments, projects, discussions, rehearsals, performances, and/or exams seemed connected to the course content and goals.*
- 88.4% students felt that *they were* **appropriately challenged**.
- 96.2% students **always** felt *supported and empowered as they worked to reach the academic goals in this course.*
- 76.9% students felt that *they* **frequently** *received feedback on assignments within a timeframe that made it useful for future work.*
- 73.0% students felt that *they found the feedback on the work they completed in this course to be* **very helpful**.
- 76.9% students **always** felt *safe, heard, and included in class discussions and activities.*
- 96.2% students **always** felt *respected, both as an individual student and as a person with their social identities.*

Before Spring 2022, OU used their standard teaching evaluation system *eValuate*.
C&AG4 = MATH 2443: Calculus and Analytic Geometry IV
LinAlg = MATH 3333: Linear Algebra I

Category	**C&AG4**	**LinAlg**	**C&AG4**
Semester	**FS2021**	**SS2021**	**FS2020**
Responses	**38/65**	**33/78**	**49/85**
Extent to which the instructor contributed to your learning*	4.63157	4.63636	4.32652
Ability of the instructor to respond to a wide range of questions about the material in this course	4.81578	4.78788	4.48979
Instructor's promptness in returning exams and assignments so they could be useful for learning	4.68421	4.54545	4.06122
Instructor's ability to encourage critical and independent thinking	4.68421	4.68750	4.30612
Instructor's ability to stimulate continuing interest in the subject matter	4.5	4.53125	4.02041
Overall instructor's teaching effectiveness was	4.73684	4.72727	4.34693
Instructor's management of the course was	4.77894	4.60606	4.38775
Amount you learned in this class*	4.5	4.36364	4.18367
Workload of this course compared to others at a similar level*	3.42105	3.51515	3.69387
Quality of readings and/or assigned course materials	4.21052	4.54545	4.02041
Overall, this course was	4.70676	4.75758	4.24489
This course was graded fairly†	4.84210	4.43750	4.79591
Instructor's preparation for class	4.78947	4.87879	4.51064
Instructor's clarity of explanation	4.63157	4.81818	4.33333
Instructor's use of examples and illustrations	4.84085	4.84848	4.52083
Instructor's skill in observing student reactions	4.71052	4.75758	4.43749
Instructor's explanation of the grading system and work requirements	4.63158	4.57576	3.77083
Instructor's availability for consultation outside class during regularly scheduled office hours	4.60526	4.84375	4.5

1 = Poor, 2 = Fair, 3 = Good, 4 = Very Good, 5 = Excellent

* 1 = Far Below Average, 2 = Below Average, 3 = Average, 4 = Above Average, 5 = Far Above Average

† 1 = Never, 2 = Seldom, 3 = Occasionally, 4 = Usually, 5 = Always

Key:	4.5-5.0	4.0-4.5	3.5-4.0	3.0-3.5	2.5-3.0	2.0-2.5	1.0-2.0

7.3. Michigan State University: Intro to Proofs

At MSU, I taught **MTH 299: Transitions (to Proofs)** on a few occasions. This page summarizes my evaluations. Please note that MSU uses an inverted scale where 1 is the best performance score and 5 is the worst.

Category	**Proofs**	**Proofs**	**Proofs**
Semester	**FS19**	**FS18**	**US18**
Responses	**15/20**	**21/25**	**8/11**
Was the instructor well-prepared for class?	1.53	1.04	1.75
How often does the instructor arrive to class on time?	1.06	1.04	1.62
The instructor provided timely feedback (graded homework, quizzes, exams, etc.).	1.26	1.14	2.00
Evaluate your instructor's willingness to help outside of class (office hours, email, etc.)†	1.2	1.19	1.5
How often did you attend the class taught by this instructor?	1.06	1.19	1.87
How interesting did you find the course material?*	1.73	1.66	1.87
Evaluate this instructor's ability to explain the course content.†	1.66	1.28	1.87
Evaluate the instructor's ability to make the course stimulating and engaging.†	1.66	1.33	1.87
Evaluate your instructor's ability to encourage questions and answer them effectively.†	1.33	1.23	1.87
Evaluate the effectiveness of this class in increasing your mathematical competency.†	1.60	1.57	1.87
Evaluate the appropriateness of the pace at which the instructor covered the material.†	1.53	1.38	1.75
Evaluate effectiveness of the homework/class assignments as a learning opportunity.†	1.73	1.57	1.75
How often did the instructor present the material in an organized manner?	1.53	1.19	1.75
Evaluate your instructor's effectiveness in communicating course information, including course policies, schedules, due dates, and grades.†	1.40	1.28	1.75
What is your overall rating of this instructor?†	1.53	1.23	2.00

1 = Almost always, 2 = Often, 3 = Sometimes, 4 = Rarely, 5 = Never
* 1 = Very interesting, 2 = Above average, 3 = Average, 4 = Below average, 5 = Not interesting at all
† 1 = Excellent, 2 = Above Average, 3 = Average, 4 = Below Average, 5 = Poor

Key:	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	3.0-3.5	3.5-4.0	4.0-5.0

At MSU, I taught all calculus courses and differential equations on a few occasions. This page summarizes my evaluations for these courses. Please note that MSU uses an inverted scale where 1 is the best performance score and 5 is the worst.

Category	**Calc I**	**Calc II**	**Calc III**	**Diff-Eq**
Semester	**US19**	**SS19**	**US17**	**US16**
Responses	**11/14**	**25/31**	**9/11**	**23/25**
Was the instructor well-prepared for class?	1.27	1.28	1.11	1.43
How often does the instructor arrive to class on time?	1.18	1.28	1.00	1.43
The instructor provided timely feedback (graded homework, quizzes, exams, etc.).	1.18	1.5	1.11	1.95
Evaluate your instructor's willingness to help outside of class (office hours, email, etc.)†	1.27	1.44	1.22	1.43
How often did you attend the class taught by this instructor?	1.36	1.44	1.37	1.26
How interesting did you find the course material?*	2.18	1.96	1.66	2.00
Evaluate this instructor's ability to explain the course content.†	1.54	1.60	1.11	1.47
Evaluate the instructor's ability to make the course stimulating and engaging.†	1.63	1.76	1.33	1.73
Evaluate your instructor's ability to encourage questions and answer them effectively.†	1.36	1.6	1.11	1.69
Evaluate the effectiveness of this class in increasing your mathematical competency.†	1.45	1.72	1.00	1.54
Evaluate the appropriateness of the pace at which the instructor covered the material.†	1.45	1.8	1.33	1.91
Evaluate effectiveness of the homework/class assignments as a learning opportunity.†	1.45	1.72	1.22	2.04
How often did the instructor present the material in an organized manner?	1.45	1.41	1.00	1.52
Evaluate your instructor's effectiveness in communicating course information, including course policies, schedules, due dates, and grades.†	1.45	1.41	1.22	1.60
What is your overall rating of this instructor?†	1.36	1.5	1.00	1.60

1 = Almost always, 2 = Often, 3 = Sometimes, 4 = Rarely, 5 = Never
* 1 = Very interesting, 2 = Above average, 3 = Average, 4 = Below average, 5 = Not interesting at all
† 1 = Excellent, 2 = Above Average, 3 = Average, 4 = Below Average, 5 = Poor

Key:	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	3.0-3.5	3.5-4.0	4.0-5.0